UNIVERSITY OF ALBERTA

# TOPICS IN SUPERSYMMETRIC ASTRO-PARTICLE PHYSICS

By

Rouzbeh Allahverdi ©

A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements for the degree of
Doctor of Philosophy

Department of Physics

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# UNIVERSITY OF ALBERTA

## FACULTY OF GRADUATE STUDIES AND RESEARCH

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# Abstract

Supersymmetry plays an important role in particle physics model building. Besides its power to solve the hierarchy problem, it opens up new possibilities to solve some of the outstanding issues in cosmology like inflation, baryogenesis, and dark matter. Consequently, most current attempts to derive realistic particle models which can address cosmological questions are being made in the context of supersymmetry. Four case studies are presented which examine issues in dark matter, inflation, and baryogenesis with a special emphasis on supersymmetry. The issues considered are stability of supersymmetric dark matter in the presence of R-parity violation in the heavy right-handed neutrino sector, the effect of final state self-interactions on parametric resonance decay of the inflaton, parametric resonance for complex fields, where the last two studies are motivated by supersymmetry, and the effect of reheating on baryogenesis from supersymmetric flat directions. The conclusions resulting from these studies range from a dependence of the dark matter decay rate on the nature of R-parity violating terms to (in principle) important modifications of simple parametric resonance in the case of supersymmetry and to substantial change in the estimates for the baryon asymmetry of the universe for a large subset of supersymmetric flat directions when only the standard model gauge symmetry is imposed.

*To my Father and my Mother*

**To my wife Mojgan ♡**

# Preface

The results in this thesis were obtained over the course of the author's Ph.D. program at the University of Alberta between 1994 and 1999. This presentation of this work is in accordance with the "Paper Format" regulations of the Faculty of Graduate Studies of the University of Alberta, and is based on the the following papers:

Chapter 2: R. Allahverdi, B. A. Campbell, and K. A. Olive, *Neutrino Mass Effects in a Minimally Extended Supersymmetric Standard Model*, Phys. Lett. **B 341** (1994), 166.

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# Chapter 1

# Introduction

## 1.1 The Electroweak Standard Model

The predictions of the standard model of elementary particles have been successfully in agreement with precision tests, so far [1]. In this model the fundamental constituents of matter are spin-1/2 fermions divided into leptons and quarks. There are spin-1 bosons, which mediate fundamental interactions between fermions, and there are spin-0 bosons, which play a technical role in the theory and have not yet been observed.

Leptons and quarks come in three generations. Leptons have only electroweak interactions that are represented by the gauge group $SU(2) \times U_Y(1)$. Left-handed leptons are $SU(2)$ doublets (one lepton and its associated neutrino) while right-handed leptons are singlets of $SU(2)$ . Quarks, in addition, participate in strong interactions with the $SU_c(3)$ gauge group. Like leptons, left-handed quarks are $SU(2)$ doublets and right-handed quarks are $SU(2)$ singlets. Both leptons and quarks carry weak hypercharge $Y$ that obeys the relation $Q = e(T_3 + \frac{Y}{2})$, where $Q$ is the electric charge and $T_3$ is the weak isospin of a particle. For a complete list of $SU(2) \times U_Y(1)$ assignments of leptons and quarks see, e.g. [2]. Each quark is also in a fundamental representation of $SU_c(3)$, something which makes anomaly cancellation possible and, therefore, the theory renormalizable. Focusing on the electroweak part of the theory, the Lagrangian consists of a kinetic part and a gauge field part. The kinetic part is

$$\begin{aligned} L^{kin} = \textstyle\sum_i (\bar{l_L}^i \gamma^\mu D_\mu l_L{}^i + \bar{e_R}^j \gamma^\mu D_\mu e_R{}^j + \bar{q_L}^i \gamma^\mu D_\mu q_L{}^i \\ + \bar{u_R}^i \gamma^\mu D_\mu u_R{}^i + \bar{d_R}^i \gamma^\mu D_\mu d_R{}^i) \end{aligned} \tag{1.1}$$

where $D_\mu = \partial_\mu - ig_2 W_{\mu,a} \frac{\sigma_a}{2} - ig_1 \frac{Y}{2} B_\mu$ for doublets and for singlets $D_\mu = \partial_\mu - ig_1 \frac{Y}{2} B_\mu$. $\vec{W}_\mu$ is the triplet of $SU(2)$ gauge bosons, $B_\mu$ is the gauge boson for hypercharge, and $g_1, g_2$ are $U_Y(1)$ and $SU(2)$ couplings respectively. The gauge field part is

$$L^G = -\frac{1}{4} W_{\mu\nu}{}^a W^{\mu\nu,a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \tag{1.2}$$

where

$$\begin{aligned} W_{\mu\nu}{}^a = \partial_\mu W_\nu{}^a - \partial_\nu W_\mu{}^a + g_2 \epsilon^{abc} W_\mu{}^b W_\nu{}^c \\ B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned} \tag{1.3}$$

We know that the electron is massive and gauge bosons that mediate the weak interaction must also be massive since this is a short range interaction. A mass term for fermions couples a left-handed spinor to a right-handed one. Such a term is not gauge invariant in the standard model because the $SU(2)$ symmetry is not vectorial. Also we can't construct a gauge invariant mass term for gauge fields in four dimensions (though it is possible in three dimensions). Gauge invariance is necessary for renormalizability of gauge theories. In order to not destroy it, explicit mass terms are forbidden in the standard model. Here the Higgs boson comes to the rescue. We can couple an $SU(2)$ doublet of spin-0 bosons with hypercharge $Y = 1$ to fermions (Yukawa couplings) to have a gauge invariant term. A non-zero vev (vacuum expectation value) for the Higgs field results in a mass term for fermions. We exploit the spontaneous symmetry breaking mechanism to get a non-zero vev for Higgs. This can be accomplished by introducing the following Higgs Lagrangian

$$L^H = D_\mu H^\dagger D^\mu H - \frac{1}{2} M^2 H^\dagger H - \frac{1}{4} \lambda (H^\dagger H)^2 \tag{1.4}$$

With $M^2 > 0$ it is easy to see that the vacuum is degenerate. Rotating the Higgs doublet $\begin{bmatrix} h^+ \\ h^0 \end{bmatrix}$ to a suitable state we choose

$$< h^0 >= \frac{M}{\sqrt{\lambda}} \ ; \ < h^+ >= 0 \tag{1.5}$$

The charge operator $Q = e(T_3 + \frac{Y}{2})$ gives zero when acting on the vacuum while the other three generators of $SU(2) \times U_Y(1)$ do not. This leads to the existence of three Goldstone bosons. These Goldstone bosons are eaten by the gauge fields $W^\pm$ and $Z$

$$\begin{gathered} W_\mu{}^\pm = \frac{W_\mu{}^1 \pm i W_\mu{}^2}{\sqrt{2}}; Z_\mu = -B_\mu \sin\theta_w + W_{\mu,3} \cos\theta_w \\ \tan\theta_w = \frac{g_1}{g_2} \end{gathered} \tag{1.6}$$

through gauge transformations. In doing so $W^\pm$ and $Z$ become massive and the $SU(2) \times U_Y(1)$ group breaks down to the $U_Q(1)$ group whose gauge field, the photon

$$A_\mu = B_\mu \cos\theta_w + W_{\mu,3} \sin\theta_w \tag{1.7}$$

remains massless. Afterwards, there is one real Higgs field with mass $M$. The most general gauge invariant form for the Yukawa couplings is

$$L^{Yuk} = h_{ij}{}^e \bar{l}_L{}^i H e_R{}^j + h_{ij}{}^d \bar{q}_L{}^i H d_R{}^j + h_{ij}{}^u \bar{q}_L{}^i \hat{H} u_R{}^j \tag{1.8}$$

where $\hat{H} = i\tau_2 H^*$ is the charge conjugate Higgs. The mass matrices $h_{ij}\frac{M}{\sqrt{\lambda}}$ can be diagonalized by unitary transformations acting on quark and lepton representations.

## 1.2 Supersymmetry

The standard model, although in very good agreement with experiment, poses many curiosities for theoretical physicists. One problem is its large number of parameters: 19 to give mass to fermions and gauge bosons, mixing angles and CP violation in the electroweak and (probably) strong sector. There is also no explanation for the number of generations and the charge quantization. GUT's could help to solve some of these [2] but a serious problem, the hierarchy problem remains. In the standard model, radiative corrections to the Higgs mass squared are quadratically divergent. This means no matter how small the Higgs mass is at tree level, it grows uncontrollably through loop orders even to the GUT or Planck scale. A Higgs mass larger than 1 TeV, however, makes the model ill-defined perturbatively [2]. In GUT's we have a similar situation. To avoid a rapid decay rate for the proton, those Higgs fields that carry baryon number must be much heavier than the usual Higgs doublet. Even if we achieve this at tree-level through fine tuning, radiative corrections still destroy it. In general, several orders of perturbation theory have to be computed for consistent fine tuning. The hierarchy problem arises in every theory that has two scales differing by a large number of orders of magnitude (like the GUT and the electroweak symmetry breaking scales). This is related to another problem, the naturalness problem. Naturalness states that if there is a parameter in the theory whose absence restores a symmetry, then perturbative corrections to that parameter are no larger than the physical value. As an example, consider the electron mass in the standard model. Gauge symmetry is unbroken when electron mass is zero. This is reflected in the fact that the radiative correction to electron mass is only logarithmic, not too large when compared with the mass itself.

For the Higgs field the situation is totally different. The standard model with a massless Higgs possesses the same symmetries as with a massive one. In this sense, the Higgs mass is not natural. There are couple of ways to avoid this problem. One is the technicolor model which assumes that the Higgs is a fermion-antifermion condensate below 1 TeV rather than a fundamental scalar. Technicolor has its own problems and it is not our aim to go into this subject here. Another suggestion is supersymmetry, a symmetry which unites fermions and bosons. In supersymmetry there is a fermionic degree of freedom for each bosonic degree of freedom (and vice versa) both with the same mass. It is easy to see why the Higgs mass is natural in supersymmetric models. With a gauge symmetry fermion masses are natural. Supersymmetry ensures massless bosons when fermions are massless. In other words, fermions protect bosons through supersymmetry. In the language of Feynman diagrams this is the miraculous cancellation of quadratic divergences in supersymmetry. Roughly speaking, for every bosonic loop there is a fermionic loop with opposite sign that cancels it. We will come back to these statements in more detail and precision.

### 1.2.1 Supersymmetry Algebra

Of all graded Lie algebras, only supersymmetry algebras generate symmetries of the S-matrix consistent with relativistic quantum field theories [3]. The proof of this statement is based on the Coleman-Mandula theorem, the most powerful in a series of no-go theorems about the possible symmetries of the S-matrix. Using this theorem, with a couple of additional assumptions, the supersymmetry algebra is found to be [3]

$$\begin{gathered}[Q_\alpha, \bar{Q}_{\dot\beta}]_+ = 2\sigma_{\alpha\dot\beta}{}^m P_m \; ; \; [P_m, Q_\alpha] = [P_m, \bar{Q}_{\dot\beta}] = 0 \\ [Q_\alpha, Q_\beta]_+ = [\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}]_+ = 0\end{gathered} \tag{1.9}$$

where $Q_\alpha$ and $\bar{Q}_{\dot\beta}$ [1] are fermionic generators which transform as $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ spinors, respectively, under the Lorentz group action.

This is the algebra of the so called $N = 1$ supersymmetry that has only one type of fermionic generators $Q$ and $\bar{Q}$ . In general, we can have any number of fermionic generators; however, our focus here is on $N = 1$ supersymmetry.

An appropriate language in which to formulate supersymmetry is the superfield language which is formulated in superspace, a set of coordinates $(x_\mu, \theta_\alpha, \bar\theta_{\dot\beta})$, where $\theta_\alpha$ and $\bar\theta_{\dot\beta}$ are Grassmann variables that satisfy the anti-commutation relations

$$[\theta_\alpha, \theta_\beta]_+ = [\bar\theta_{\dot\alpha}, \bar\theta_{\dot\beta}]_+ = [\theta_\alpha, \bar\theta_{\dot\beta}]_+ = 0 \tag{1.10}$$

---

[1] for a review on spinor notation and conventions see, e.g. [4].

The supersymmetry algebra can then be rewritten as

$$\begin{aligned} [\theta Q, \bar{Q}\bar{\theta}] &= 2\theta\sigma_{\mu}\bar{\theta}P^{\mu} \\ [\theta Q, \theta Q] &= [\bar{Q}\bar{\theta}, \bar{Q}\bar{\theta}] = 0 \end{aligned} \tag{1.11}$$

with spinor indices suppressed for simplicity, and a supersymmetry transformation written as

$$S(x, \theta, \bar{\theta}) = \exp[i(\theta Q + \bar{Q}\bar{\theta} - x_{\mu}P^{\mu})] \tag{1.12}$$

It is shown in [3] that a representation of the supersymmetry algebra in terms of differential operators

$$P_{\mu} = i\partial_{\mu} \; ; \; Q = \partial_{\theta} - i\sigma^{\mu}\bar{\theta}\partial_{\mu} \; ; \; \bar{Q} = -\partial_{\theta} + i\theta\sigma^{\mu}\partial_{\mu} \tag{1.13}$$

can be derived, and covariant derivatives that anticommute by which infinitesimal supersymmetry transformations are given by

$$D = \partial_{\theta} + i\sigma^{\mu}\bar{\theta}\partial_{\mu} \; ; \; \bar{D} = -\partial_{\theta} - i\theta\sigma^{\mu}\partial_{\mu} \tag{1.14}$$

There are also L- and R-representations of the supersymmetry algebra [3] where

$$\begin{aligned} Q_L &= \partial_{\theta} \; ; \; \bar{Q}_L = -\partial_{\theta} + 2i\theta\sigma^{\mu}\partial_{\mu} \\ Q_R &= \partial_{\theta} - 2i\sigma^{\mu}\bar{\theta}\partial_{\mu} \; ; \; \bar{Q}_R = -\partial_{\theta} \end{aligned} \tag{1.15}$$

with the corresponding covariant derivatives

$$\begin{aligned} D_L &= \partial_{\theta} + 2i\sigma^{\mu}\bar{\theta}\partial_{\mu} \; ; \; \bar{D}_L = -\partial_{\theta} \\ D_R &= \partial_{\theta} \; ; \; \bar{D}_R = -\partial_{\theta} - 2i\theta\sigma^{\mu}\partial_{\mu} \end{aligned} \tag{1.16}$$

### 1.2.2 Superfields

A superfield $\Phi(x, \theta, \bar{\theta})$ is a function that transforms as follows under the supersymmetry transformation $S(y, \alpha, \bar{\alpha}) = \exp[i(\alpha Q + \bar{Q}\bar{\alpha} - y_{\mu}P^{\mu})]$

$$\Phi(x, \theta, \bar{\theta}) \rightarrow \Phi(x + y - i\alpha\sigma_{\mu}\bar{\theta} + i\theta\sigma_{\mu}\bar{\alpha}, \theta + \alpha, \bar{\theta} + \bar{\alpha}) \tag{1.17}$$

Consider the Taylor expansion of a superfield in terms of $\theta$ and $\bar{\theta}$ . Because $\theta$ and $\bar{\theta}$ are Grassmann variables, the series is terminated at the $\theta\theta\bar{\theta}\bar{\theta}$ term. A (left-) right-handed chiral superfield is a superfield that satisfies $(\bar{D}\Phi)$ $D\Phi = 0$. In the L-representation of the supersymmetry algebra

$$\bar{D}_L = -\partial_{\theta} \tag{1.18}$$

so a left-handed chiral superfield in the L-representation is a function only of $\theta$ and can be written as follows

$$\Phi_L(x,\theta) = \phi(x) + \theta\psi(x) + \theta\theta F(x) \tag{1.19}$$

with spinor indices suppressed for simplicity. $\phi$ and $F$ are complex scalars and $\psi$ is a left-handed Weyl spinor. They are called component fields of the superfield $\Phi$ . The variation of these component fields under a supersymmetry transformation is

$$\delta\phi = \sqrt{2}\alpha\psi; \delta\psi = \sqrt{2}\alpha F + i\sqrt{2}\sigma^{\mu}\bar{\alpha}\partial_{\mu}\phi; \delta F = -i\sqrt{2}\partial_{\mu}\psi\sigma^{\mu}\bar{\alpha} \tag{1.20}$$

We notice that the variation of the highest component $(F)$ of a chiral superfield under a supersymmetry transformation is a total derivative. Analogously, a right-handed chiral superfield in the R-representation (where $D_R = \partial_{\theta}$) can be written as follows

$$\Phi_R(x,\bar{\theta}) = \phi(x) + \bar{\theta}\psi(x) + \bar{\theta}\bar{\theta}F(x), \tag{1.21}$$

where $\psi$ is a right-handed Weyl spinor. It is seen that conjugate ${\Phi_L}^{\dagger}$ of a left-handed chiral superfield $\Phi_L$ in the L-representation is a right-handed chiral superfield in the R-representation. To bring them into the same representation we have to do the following replacement [3]

$$\Phi_R(x,\theta,\bar{\theta}) = \Phi_L(x - 2i\theta\sigma_{\mu}\bar{\theta},\theta) \tag{1.22}$$

where $\Phi_R$ is now a right-handed chiral superfield in the L-representation. The product of any number of left- (right-)handed chiral superfields is a left- (right-)handed chiral superfield. There are also vector superfields, superfields which are real, i.e. $V = V^\dagger$ . Vector superfields are functions of both $\theta$ and $\bar{\theta}$ and in the Wess-Zumino gauge are written as follows [3]

$$V(x) = -\theta\sigma^\mu\bar{\theta}V_\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x) \tag{1.23}$$

where $V_\mu$ is a vector field, $\lambda$ is a left-handed Weyl spinor, and $D$ is a real scalar field. Variation of these component fields under supersymmetry transformation is

$$\delta\lambda = \alpha\sigma^{\mu\nu}V_{\mu\nu}; \delta V_\mu = i\alpha\sigma_\mu\bar{\lambda} + i\bar{\alpha}\bar{\sigma}_\mu\lambda; \delta D = -\alpha\sigma^\mu\partial_\mu\bar{\lambda} + \bar{\alpha}\sigma^\mu\partial_\mu\lambda \tag{1.24}$$

where $\sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)$, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ . It is seen that the variation of the highest component ($D$) of a vector superfield is also a total derivative. The product of an equal number of left- and right-handed chiral superfields is a vector superfield.

We notice that both chiral and vector superfields contain an equal number of bosonic and fermionic degrees of freedom. In a chiral superfield, $\psi$ provides four fermionic degrees of freedom before using the Dirac equation, while $\phi$ and $F$ each provide two bosonic degrees of freedom, for a total of four. In a vector superfield, $\lambda$ provides four fermionic degrees of freedom (again, before using the Dirac equation), while $V$ provides three and $D$ another bosonic degree of freedom.

### 1.2.3 Supersymmetry Lagrangian

As mentioned earlier, the variation of F- and D-terms under supersymmetry transformation is a total derivative. This gives us a clue for supersymmetry model building. Integration of a (chiral) vector superfield over superspace coordinates leaves us with the integral of (F-)D-term over spacetime, which is invariant not only under Lorentz transformation, but also under supersymmetry transformation.

As a simple example we consider supersymmetric version of the $\lambda\phi^4$ theory. With a left-handed chiral superfield

$$\Phi(x) = \phi(x) + \theta\psi(x) + \theta\theta F(x) \tag{1.25}$$

mass terms and interactions are derived from the F-term (superpotential)

$$L_F = \frac{1}{2}m\Phi^2 + \frac{1}{3}\lambda\Phi^3 \tag{1.26}$$

while kinetic terms are added through the D-term

$$L_D = \frac{1}{2}\Phi^\dagger\Phi \tag{1.27}$$

The Lagrangian density of the theory is

$$L = \int d^2\theta d^2\bar{\theta} L_D + \int d^2\theta L_F + h.c. \tag{1.28}$$

The $F$ field is an auxiliary field which can be eliminated through equations of motion

$$F = -m\phi^* - \lambda\phi^{*2} \tag{1.29}$$

In general, given a superpotential $W(\Phi_i)$, we can find the scalar potential arising from it

$$V = \left| \frac{\partial W(\phi_i)}{\partial\phi_i} \right|^2 \tag{1.30}$$

Supersymmetric version of a gauge field strength is derived from a F-term. With the vector superfield $V$

$$V(x) = -\theta\sigma_\mu\bar{\theta}V^\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x) \tag{1.31}$$

in the Wess-Zumino gauge [3], it turns out that $\frac{1}{32}(W^\alpha W_\alpha)$ gives the desired term.

In the abelian case

$$W_\alpha = \bar{D}\bar{D}D_\alpha V \tag{1.32}$$

while for the non-abelian case

$$W_\alpha = \bar{D}\bar{D}[exp(-gV)D_\alpha exp(gV)] \tag{1.33}$$

where $g$ is the gauge coupling, $V_\mu = V_\mu{}^a T^a$, and $T^a$ are generators of the non-abelian gauge group. The $D$ field is an auxiliary field which can be eliminated through equations of motion.

Finally, the coupling of the gauge field to matter is through the D-term

$$[\Phi^\dagger exp(2gV)\Phi]_D \tag{1.34}$$

where $\Phi$ is in some representation of gauge group [3]. For a detailed look at the Lagrangian of a supersymmetric gauge theory see, e.g. [4].

### 1.2.4 Supersymmetry Breaking

As long as supersymmetry is exact, there is no renormalization of the superpotential, either finite or infinite, in perturbation theory. Therefore, if , for some reason, a fine tuning happens at tree-level, it will be preserved to any order of perturbation theory. This is the famous non-renormalization theorem for supersymmetry. It ensures that if supersymmetry is unbroken at tree-level, it is unbroken at any order of perturbation theory. So there is no analogue to the Coleman-Weinberg mechanism for supersymmetry breaking. In exact supersymmetry, all renormalization is in D-terms and divergences are absorbed in wavefunctions and gauge couplings. Furthermore, there are no quadratic divergences, only logarithmic ones. All the statements we made can readily be verified by using the superfield formalism [5]. This is the reason behind our hope to render the standard model to a natural theory by making it supersymmetric. However, if supersymmetry has anything to do with particle physics, it must be broken at low energies. We do not observe equal mass superpartners of the standard model particles.

In the case of spontaneous supersymmetry breaking the vacuum is not invariant under supersymmetry transformations. We recall that the variation of a chiral superfield under such a transformation is

$$\delta\phi = \sqrt{2}\alpha\psi \; ; \; \delta\psi = \sqrt{2}\alpha F + i\sqrt{2}\sigma^{\mu}\bar{\alpha}\partial_{\mu}\phi \; : \; \delta F = -i\sqrt{2}\partial_{\mu}\psi\sigma^{\mu}\bar{\alpha} \qquad (1.35)$$

We want the Lorentz transformation symmetry to be unbroken, so the vev of $\psi$ must be zero. Also $\partial_{\mu}\phi = 0$ in the ground state: consequently $\delta\phi = \delta F = 0$. $\delta\psi$ is non-zero only if $< F > \neq 0$. A massless fermion, the Goldstino, arises in spontaneous breakdown of supersymmetry through a non-zero vev for the F-term, the F-type breaking. It is also possible that supersymmetry breaking happens through a non-zero vev for a D-term, the D-type breaking, in which case there also exists a Goldstino.

There are no quadratic divergences and only finite renormalization of the superpotential for the F-type (and also upon satisfying certain conditions for the D-type) breaking [5]. However, there are some phenomenological problems which arise when one attempts to build realistic particle physics models based on the F-type breaking (O'Rafeartaigh model) or the D-type breaking (Fayet-Illiopoulos model) [6]. Today, the most common scenario for producing low energy supersymmetry breaking is the hidden-sector scenario. In this scenario, supersymmetry breaks in a "hidden sector" which interacts only through gravity or gauge interactions with ordinary matter, at either high (e.g. $10^{11}$ GeV) or medium (e.g. $10^{4}$ GeV) scales, depending on the type of mediation, and it manifests itself in explicit supersymmetry breaking terms which are continued through renormalization group equations to low energies. These are called soft supersymmetry breaking terms, taking the form

$$\tilde{M}_1 Re(\phi^2) + \tilde{M}_2 Im(\phi^2) + C(\phi^3 + h.c.) + \tilde{M}_3(\lambda^a\lambda^a + \bar{\lambda}^a\bar{\lambda}^a) \tag{1.36}$$

where $\phi$ is a scalar field, and $\lambda^a$ is a gaugino (notice that all terms are gauge invariant combinations). These terms lead to logarithmic divergences and all divergences are absorbed in wavefunctions, gauge couplings and soft supersymmetry breaking terms.

### 1.2.5 Supersymmetry and Particle Physics

Supersymmetry, because of its potential to solve the hierarchy problem, plays an important role in particle model building. Besides that, there are other hints that tell us that supersymmetry may be relevant. Among them we can name the natural emergence of gravity from local supersymmetry and viability of the unification of the coupling constants of the standard model within the context of supersymmetry. The most important problem which remains to be solved is the mechanism of supersymmetry breaking and its manifestation at low energies. Here we consider the supersymmetric version of the standard model, the MSSM (minimal supersymmetric standard model), which contains no new particles except for the superpartners of the standard model particles and a new (doublet) Higgs superfield which is needed for technical reasons [2]. The standard model particles and their superpartners belong to the same representation and have the same quantum numbers . The fermionic partner of the standard model Higgs is a doublet with $Y = 1$. For anomaly cancellation, another fermionic doublet (and its bosonic partner) with $Y = -1$ is needed. This is the origin of the new Higgs superfield. This new Higgs is also needed to give mass to the top quark. In supersymmetry, Yukawa couplings come from the superpotential and the superpotential is an analytic function of the superfields. Therefore, the charge conjugate of the standard model Higgs cannot be used to give mass to the top quark; we have to introduce a new Higgs with quantum numbers which are charge conjugate of the standard model Higgs.

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[2] It is easily figured out that none of the superpartners of the standard model particles are contained in the standard model [6].

The superpotential of the MSSM can be written as the sum of Yukawa couplings (with generation indices suppressed)

$$F_Y = h_u H_1 Q u^c + h_d H_2 Q d^c + h_e H_2 L e^c \tag{1.37}$$

where $H_1$ and $H_2$ are two Higgs doublets, $Q$ and $L$ are doublets of left-handed quarks and leptons respectively, $u^c$ the left-handed anti-up quark, $d^c$ the left-handed anti-down quark, and $e^c$ the left-handed anti-electron, all superfields. It is interesting to see what global phase symmetries this superpotential can have. There are two types of such symmetries: those which commute with supersymmetry and those which do not. In the first type, all component fields of a superfield transform identically under the action of the $U(1)$ symmetry group. It is easy to show that there are four symmetries of this type for $F_Y$ : lepton number, baryon number, hypercharge and PQ symmetry [7], where all but hypercharge are anomalous.

The second type of global phase symmetries are called R-symmetries, under which $\theta$ coordinates also transform. This means that the component fields of a superfield transform differently under the action of $U(1)$ group. $F_Y$ also has an R-symmetry [7]. R-symmetry does not seem to be a symmetry of nature, therefore, if not explicitly, it must be broken spontaneously. R-symmetry is anomalous, which leads to the emergence of a pseudo-Goldstone boson, the R-axion. This is problematic from an experimental point of view. R-symmetry also forbids gauginos from having mass, which is viable.

If we add the gauge singlet term $\mu H_1 H_2$ to $F_Y$, both PQ and R-symmetry break explicitly. However the linear combination $\frac{1}{3}$PQ+R is still preserved and anomalous; hence the same problem arises. By adding soft supersymmetry breaking terms

$${m_1}^2|H_1|^2 + {m_2}^2|H_2|^2 + m_u h_u H_1 \tilde{Q} \tilde{u}^c + m_d h_d H_2 \tilde{Q} \tilde{d}^c + m_e h_e H_2 \tilde{L} \tilde{e}^c + m_6 \mu H_1 H_2 \tag{1.38}$$

this symmetry breaks to a $Z_2$ discrete symmetry called R-parity. R-parity assigns the factor $(-1)^{L+3B+2S}$ to each particle where $L$, $B$, and $S$ are lepton number, baryon number, and spin of the particle, respectively. Eventually, the following superpotential

$$F = F_Y + F_H = h_u H_1 Q u^c + h_d H_2 Q d^c + h_e H_2 L e^c + \mu H_1 H_2 \tag{1.39}$$

along with the above soft supersymmetry breaking terms, the supersymmetry breaking gaugino mass terms, the associated $D$-terms, the kinetic terms, and the gauge terms constitute the MSSM Lagrangian. Lagrangians for the extensions of the MSSM (e.g. supersymmetric GUT's) can be built accordingly by imposing the gauge invariance and the fact that superpotential is an analytic function of superfields. For a good review on prospects of supersymmetry for particle model building see, for example [8].

## 1.3 Cosmology

The universe, though presently old and large, is believed to have been very small in its early stages. This suggests that particle physics played a role in dynamics of the universe since the very beginning. Actually, there is a strong interplay between particle physics and cosmology: particle theory could explain some of the cosmological phenomena, while cosmology strongly restricts particle physics model building. The early universe, because it was in thermal equilibrium, is a laboratory for testing new physics at high energies unreachable on earth.

### 1.3.1 Big Bang Cosmology

The standard big bang cosmology is a successful model that explains some observational aspects of the universe very elegantly. The big bang model is based on three theoretical pillars: the Einstein equations, the cosmological principle, and the perfect fluid description of matter. Together, these give the following metric

$$ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)] \tag{1.40}$$

for the universe when using spherical coordinates [9]. The constant $k$ is +1, 0, or -1 for a closed, flat and open universes, respectively. $a(t)$ is the scale factor of the universe and its evolution depends on the phase of the perfect fluid (i.e. matter, radiation, ...) . The universe is matter-dominated today, but at early stages most of its energy was in the form of radiation. There are three observational pillars for the big bang model: the observed redshift of galaxies, the CMBR (cosmic microwave background radiation), and the light element abundances, all of which are in very good agreement with theory [9].

The CMBR provides a snapshot of the universe at the moment of recombination, when atoms first became stable. Photons, which had a very short mean free path in the ionized matter before recombination, interact for the last time and reach us from the surface of last scattering. This represents the best example of black-body radiation experimentally allowed with deviation of less than 0.005% and a temperature $T$ = 2.7277K measured by COBE [10]. Predictions of BBN (big bang nucleosynthesis) for the abundance of $^4$He also strongly support the big bang model. The primordial abundance of $^4$He depends on three factors: the neutron lifetime, the number of light neutrinos (light enough to be relativistic at $T$ = 1 MeV, when the weak interaction freezes), and the ratio of the density of baryons to the density of photons [9]. Recent measurements of the $^4$He abundance point to a $Y = \frac{^4He}{H}$ in the range 0.235 – 0.245 [11, 12]. For a recent review on BBN see, for example [13].

### 1.3.2 Inflation

There are some shortcomings in the big bang model which, while not in contradiction with the theory, nevertheless require fine tuning and very special initial conditions.

The most famous of these are the flatness-entropy and horizon problems; depending on the particle physics model of the early universe, there could be monopole and gravitino problems as well [14]. At present, the only solution to all of these problems is inflation [14] [3]. During inflation, the universe experiences a period of superluminal expansion which stretches a small patch within the horizon into a very large volume flattened to a very high degree. A scalar field, called the inflaton, is needed to drive inflation. The inflaton energy density is almost constant during inflation, while all other kinds of energy density will be inflated away. In other words, the pre-inflation memory of the universe is erased. Some 60 e-folds of inflation would be enough to solve both the flatness and horizon problems. In order to have successful inflation, the inflaton scalar potential must be very flat for some range of the field value. Unfortunately there is to date no realistic model for inflation from the particle perspective.

At the end of inflation, the inflaton enters an oscillatory regime, during which the energy density of the universe is dominated by the coherent oscillations of the inflaton field, which behave like non-relativistic matter. The inflaton will efficiently decay to those particles to which it is coupled when the decay rate is equal to or larger than the Hubble constant. Upon thermalization of decay products, we have the familiar radiation-dominated FRW universe and the dynamics will be that of the big bang model from then on. The transition from the former to the latter one is called reheating, the details of which are of great importance. Until recently, it was thought that inflaton decay occurs in the perturbative regime through one-particle decay, allowing a simple estimate for the reheat temperature [16]. In the past five years it has been noted that the dynamics of decay is generally much more complicated. For specific ranges of the inflaton coupling and its initial amplitude, the decay rate is substantially amplified and decay products are explosively produced in certain momentum bands (particularly for bosons, for which occupation numbers larger than one are allowed). Non-perturbative effects can become very important, leading to a very rapid decay of the inflaton called parametric resonance decay (for a detailed discussion see [17]).

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[3] Although there are other suggestions to solve some of these problems, inflation looks as if it is the only candidate to solve all [15].

Close to the end of this stage, the decay products have a non-thermal momentum distribution with energies typically much higher than the inflaton mass. At this stage, very interesting and novel phenomena can occur like non-thermal symmetry restoration with subsequent formation of topological defects [18, 19], production of superheavy particles even with masses close to the Planck mass [20, 21], strong supersymmetry breaking [22], and gravitino production [23]. Consequently, all viable inflationary models must be reconsidered to see whether the inflaton decay is indeed via resonance and (in this case) leads to cosmologically unacceptable implications.

### 1.3.3 Baryogenesis

All the matter we observe today is made out of baryons, almost all of them nucleons. The question is: what causes such an asymmetry between baryons and antibaryons? It could be an initial condition, but then we need a fine tuning which is not desirable. In addition, with a period of inflation, any such baryon asymmetry will be inflated away. Another possibility is that after the freeze-out of nucleon-antinucleon annihilation, matter and antimatter were separated somehow, so that there are parts of the universe which fully consist of antibaryons, just as our part consists of baryons. This suggestion has serious problems. It does not give the correct amount of baryons which we observe today and apparently violates causality [14].

It was Sakharov [24] who first pointed out the necessary ingredients to produce a baryon asymmetry from symmetric initial conditions:

1- Baryon number violating interactions.

2- C and CP violation.

3- Out of thermal equilibrium condition.

It was understood later that GUT's could provide these conditions. In GUT's, there are heavy gauge and Higgs bosons that carry baryon number [2], CP violation is provided through phases in fermion Yukawas, and the out of equilibrium condition is satisfied when the temperature of the early universe drops below the gauge (Higgs) boson mass. Lower bounds on the mass of heavy gauge bosons can be found from laboratory bounds on baryon number violating interactions like proton decay. However, in the simplest GUT's (e.g. $SU(5)$) which preserve $B-L$, it is easy to show that if electroweak interactions are in thermal equilibrium, any baryon asymmetry is eventually washed out [25]. There are other suggestions to produce the BAU (baryon asymmetry of the universe). It can be shown [26] that with some conserved number and asymmetry among generations, a net baryon number is generated after the fermions acquire mass, even if one starts initially with $B-L=0$. It was also suggested [27] that an asymmetry would be preserved if some of it was carried by right-handed fermions, as long as some of the Yukawa couplings were out of equilibrium.

The out of equilibrium decay of gauge bosons scenario does not seem viable in the presence of inflation. The reheat temperature in inflationary models is generically of the order $10^{10}-10^{11}$ GeV, for perturbative inflaton decay. In $SU(5)$, for example, mass of the heavy gauge boson is of the order $10^{15}$ GeV. Therefore, there would not be enough gauge bosons after reheating to produce the observed BAU. The situation for the heavy Higgs is somewhat better, as their mass is typically four orders of magnitude smaller than the mass of heavy gauge bosons [25].

There are other mechanisms by which one may generate a baryon asymmetry without GUT's. One uses the baryon number anomaly of the standard model as its basic ingredient. The $SU(2)$ vacuum is not unique, it consists of topologically distinct sectors with their assigned Chern indices [28]. Going from one sector to another changes both the baryon and lepton number, but preserves their difference $B-L$. At zero temperature, the only way for the transition to take place is through tunneling, the so-called instanton solution. At finite temperature, sphalerons mediate the transition, and above the electroweak breaking scale, the transition rate is very fast. Sphalerons, CP-violating phases in the CKM matrix in the standard model, and the electroweak phase transition (if first order) provide all the necessary ingredients pointed out by Sakharov, and could result in the baryon asymmetry we observe today [29]. However, this mechanism is insufficient to lead to the observed BAU within the context of the standard model [30].

There is another alternative which uses the sphaleron effect. A net lepton number asymmetry can be generated if lepton number violating interactions that violate C and CP become out of equilibrium. In a simple model [31], three heavy right-handed neutrinos with both Dirac and Majorana mass terms are added to the standard model. Lepton number is violated by the Majorana mass term and CP violating phases can arise in Yukawa couplings. Out of equilibrium decay of heavy neutrinos generates a net lepton number in the light sector that is partially converted to baryon number through sphalerons.

For a recent review on theories of baryogenesis see, for example [30].

### 1.3.4 Dark Matter

Based upon BBN, concordance of light element primordial abundances requires [9]

$$4(3) \times 10^{-10} < \eta < 7(10) \times 10^{-10} \tag{1.41}$$

where $\eta = \frac{n_B}{n_\gamma}$, with $n_B$ and $n_\gamma$ the baryon and photon number density respectively. This means

$$0.015(0.011) \leq \Omega_B h^2 \leq 0.026(0.037) \tag{1.42}$$

where $H = 100h$ and $\Omega_B = \frac{8\pi G}{3}\frac{\rho_B}{H^2}$, with $H$ the Hubble constant and $\rho_B$ the baryon energy density. For a generous range in the hubble constant

$$0.015(0.011) \leq \Omega_B \leq 0.16(0.21) \tag{1.43}$$

The Einstein equations give

$$H^2 = \frac{8\pi}{3}G\rho - \frac{k}{a^2} \tag{1.44}$$

We define $\rho_c = \frac{3}{8\pi G}H^2$ as the critical density and $\Omega = \frac{\rho}{\rho_c}$, where $\rho = \rho_M + \rho_\Lambda$, with $\rho_M$ and $\rho_\Lambda$ denoting energy density in matter and cosmological constant, respectively. We notice that for a flat ($k = 0$) universe $\Omega = 1$ . The inflationary scenario also predicts $\Omega = 1$, otherwise fine tuning will be needed.

That portion of matter which we observe in our galaxy (all baryonic) accounts for only $\Omega \simeq 0.01$ . The flatness of the rotational curve of the galaxy beyond its disk suggests that (at least) $\Omega \simeq 0.1$ for the galaxy. This provides evidence which suggests that the gravitationally dominant mass component of the universe is dark, i.e. it is not seen either in emission or absorption of any type of electromagnetic radiation. This is the dark matter (DM). Recent measurements give $\Omega_{DM} > 0.15$ (an absolute lower bound, coming from satellites of spiral galaxies), $\Omega_{DM} = (0.19 \pm 0.06)$ (from the mass over light ratio of clusters), $\Omega_{DM} = (0.44 \pm 0.11)$ (from the baryon fraction in clusters, using BBN), $\Omega_{DM} = (0.55 \pm 0.17)$ (from the abundance of high-$z$ clusters). There are many candidates for dark matter in particle theories. For the baryonic case see [32]. The major candidates for non-baryonic dark matter are:

1- Light neutrinos (hot dark matter): if one or more standard neutrino species are non-relativistic at present, then their contribution to the present density of the universe is $\rho_\nu = \sum_i m_{\nu i}(n_{\nu i} + n_{\bar{\nu}_i}) = \Omega_\nu \rho_c$, thus

$$\Omega_\nu h^2 = \frac{\sum_i m_{\nu i}}{92\text{eV}} \tag{1.45}$$

Estimates based on the density of $^2$D [33], whose primordial abundance is the best known among the light elements [34], give $\Omega_B = (0.019 \pm 0.0024)h^{-2}$, comparable to the density of a standard 2 eV neutrino.

2- Axions (cold dark matter): this is a pseudo-Goldstone boson that appears in the PQ (Peccei-Quinn) solution to the strong CP problem. An axion with the mass

$10^{-5}$ eV could close the universe. For a good review on axions see, for example [35].

3- LSP (lightest supersymmetric particle): a cold dark matter candidate in supersymmetric models. We will come back to this in more detail later on.

4- Cosmological constant: during last two years two new pieces of strong evidence for $\Omega_M < 1$ have appeared. The first of them is based on the evolution of the abundance of rich galaxy clusters with redshift $z$ [36]. The second, completely independent, argument for $\Omega_M = (0.2 - 0.4)$ follows from direct observations of supernovae (type Ia) explosions at redshifts up to $z \sim 1$ [37]. For a good discussion in this regard see, for example [38].

### 1.3.5 Supersymmetry and Cosmology

Supersymmetry could possibly answer some basic questions in cosmology, such as the origin of dark matter and the BAU, yet at the same time it could also impose some restrictions. As an example, consider inflation in the context of supersymmetry. As mentioned earlier, the superpotential gets only a finite renormalization from radiative corrections when supersymmetry is softly broken. That part of the potential which comes from D-terms

$$V = \frac{1}{2} D^a D^a \; ; \; D^a = g \phi_i^* T^a{}_{ij} \phi_j \qquad (1.46)$$

has logarithmic divergences, but it is zero when $\phi$ is a gauge singlet. It suggests that for a singlet inflaton we can construct a flat potential at tree-level without worrying about radiative corrections. On the other hand, the reheat temperature in supersymmetric models must be lower than $\sim 10^{10}$ GeV. The reason for this constraint is the presence of gravitinos, superpartners of gravitons. They are so weakly coupled to other particles that if they are produced at higher temperatures, they can't decay rapidly enough and eventually dominate the energy density of the universe, destroying the predictions of BBN [14].

The MSSM introduces a natural candidate for dark matter as a result of its discrete symmetry, R-parity. Under R-parity all standard model particles are assigned 1, while all their superpartners are assigned -1. R-parity conservation means that decay products of a supersymmetric particle must include an odd number of supersymmetric particles. So the lightest supersymmetric particle (LSP) is stable as long as R-parity is conserved. The question is: what is the LSP? It is likely to be electrically neutral and have only weak interactions. If it had either electric charge or strong interactions, it would presumably have lost energy and condensed into the galactic disk along with ordinary matter. In this case, it would be detectable as an anomalous heavy isotope of ordinary matter, in conflict with experimental limits. It also occurs rarely in models to have a coloured or charged LSP, naively because mass of such a LSP is more sensitive to radiative corrections.

The neutral and colourless candidates in the MSSM are the gravitino, the sneutrino, two neutral Higgs fermions (Higgsinos), and two neutral gauge fermions (gauginos). In models with gauge mediated supersymmetry breaking, the gravitino is the LSP. In gravity mediated models, however, its cosmological relevance would require inflation with just the right amount of reheating. Sneutrinos have also been ruled out by experiments on direct dark matter detection. The remaining candidates are neutral gauginos ($\tilde{W}^3, \tilde{B}$) and Higgsinos ($\tilde{H}_1^0, \tilde{H}_2^0$), known as neutralinos. The LSP is the linear combination of neutralinos which has the lowest mass. There are allowed regions of parameter space of the MSSM for the LSP that are specified by dark matter considerations. In special cases the LSP is a pure photino, bino or some combination of Higgsinos [32].

Supersymmetry also provides an interesting alternative for baryogenesis, the so-called Affleck-Dine mechanism [39]. In the limit of unbroken supersymmetry, the ground state of theory has many flat directions. Soft supersymmetry breaking terms slightly lift this large vacuum degeneracy. If there are effective terms in the potential that violate baryon number (these terms could come from a GUT) plus CP-violating phases, an initial value of a scalar field along a flat direction [4] can evolve to a net baryon number that is conserved in sfermions during the expansion of early universe [39]. After reheating and decay of the flat direction this baryon number is carried both by fermions and sfermions and eventually leads to the observed BAU.

In the original model, however, an $SU(5)$ GUT was used to provide the baryon number violating operator [39]. In $SU(5)$ $B-L$ is conserved and, as we mentioned earlier, if all interactions are in equilibrium, then sphalerons wash out any baryon and lepton number and finally $B = L = 0$ . It was already pointed out in the original paper [39] that Bose-Einstein condensates of sfermions may form after reheating. If most of the baryon number is carried by the condensates, it is safe from erasure by sphalerons. During expansion of the universe, both the critical temperature for condensate formation $T_c$ and the temperature of universe $T$ drop, the former proportional to $a^{-\frac{3}{2}}$ the latter to $a^{-1}$ [40], where $a$ is scale factor of the universe. If $T_c$ does not drop below $T$ until sphalerons go out of equilibrium, the baryon number is safe from erasure. After condensate evaporation there is no longer a sphaleron effect, and part of the baryon number carried by fermions leads to the BAU.

There are also models [41, 42] that use the same features, flat directions in the ground state and scalars that carry lepton and baryon number, to derive a lepton asymmetry which is partially converted to a baryon asymmetry through sphalerons. These models contain right-handed neutrinos and use lepton number violating operators to get a net lepton number carried by sleptons and, after reheating, by leptons. In these models $B-L \neq 0$ initially, so there is no need to require the formation of Bose-Einstein condensates of sfermions.

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[4] This initial value can be provided by quantum fluctuations during inflation.

## 1.4 Onwards

In this thesis, the reader will find four relatively self-contained investigations dealing with topics in astro-particle physics, with a special emphasis on supersymmetry. In chapter 2, a minimal extension of the MSSM, including three (heavy) right-handed (s)neutrinos to accomodate neutrino masses via the see-saw mechanism, is considered. R-parity violating terms in the heavy neutrino sector are introduced (there exists already strict laboratory and cosmological bounds on direct R-parity violation in the low energy sector [43]). The induced R-parity violation in the MSSM sector will be investigated and the instability of the LSP as the dark matter candidate will be used to restrict high energy R-parity violating terms.

Chaper 3 deals with the resonance decay of the inflaton to final state bosons with self-interactions of gauge strength. It is well known that gauge non-singlet bosons in supersymmetry have such self-interactions coming from the D-term part of the scalar potential. It will be shown that parametric resonance is substantially modifed in the presence of self-interactions, which could possibly avoid disastrous overproduction of gravitinos. In chapter 4, motivated by the fact that supersymmetry inherently deals with complex scalar fields, parametric resonance for a complex oscillating scalar field with phase invariant coupling to the final state fields is investigated. It will be shown that an out of phase component in the oscillations of the real and imaginary parts of the oscillating field could kill the resonance in some cases. The effect of such an out of phase component on some alternative mechanisms of explosive particle production will be studied, as well as its effect on parametric resonance in an expanding universe.

In chapter 5 baryo/lepto-genesis from the oscillation of condensates along flat directions of the supersymmetric standard model, which attained large vevs at the end of the inflationary epoch, will be examined. The key observation is that superpotential interactions couple the flat directions to other fields whose induced mass from the flat direction vev may be sufficiently small that they are kinematically accessible to inflaton decay. In such cases the flat directions start their oscillations at an earlier time than usually estimated; the oscillations are also terminated earlier, due to evaporation of the flat direction condensate produced by its interaction with the plasma of inflaton decay products. In these cases we find that estimates for the resulting baryon/lepton asymmetry of the universe are substantially altered.

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# Chapter 2

# Neutrino Mass Effects in a Minimally Extended Supersymmetric Standard Model

## 2.1 Introduction

Despite its current experimental elusiveness, the supersymmetric standard model remains theoretically well motivated. Supersymmetry itself represents the unique possibility to combine internal and spacetime symmetries in quantum field theories, evading "no go" theorems by its incorporation of anticommutation relations in its defining algebraic structure. The gauging of supersymmetry inevitably results in a general coordinate invariant theory, and Einstein gravity, and leads along the path of unification of gravity with the gauge interactions. Unification of the gauge interactions themselves appears to be facilitated by supersymmetry; extrapolation of the gauge coupling constants of the standard model gauge group factors according to the renormalization group equation with standard model matter content does not result in them coming together at a single point. On the other hand, when the superpartners of the standard model matter multiplets (including two Higgs doublets as required in the supersymmetric standard model) are included in the renormalization group running above the electroweak scale then the coupling constants for SU(3), SU(2), and U(1), cross at an energy scale of order $10^{16}$ GeV [1], as would be required for unification, and this unification scale is consistent, in these theories, with the observed stability of the proton. Finally, the inclusion of supersymmetry partners at about the electroweak scale is essential for the strongest phenomenological motivation for supersymmetry, which is to explain the stability of the electroweak scale under radiative corrections, and the maintenance of the hierarchy between the electroweak scale and the GUT or Planck scales.

It is well known, however, that the minimal supersymmetric standard model (MSSM) contains only a few of the possible gauge invariant couplings. With a minimal field content, the superpotential can be written as the sum of Yukawa terms (with generation indices suppressed)

$$F_Y = h_u H_1 Q u^c + h_d H_2 Q d^c + h_e H_2 L e^c \tag{2.1}$$

and to avoid an axion like state a mass term mixing the two Higgs doublets $H_1$ and $H_2$,

$$F_H = \mu H_1 H_2 \tag{2.2}$$

$Q$ and $L$ are weak doublets and $u^c, d^c$, and $e^c$ are their corresponding right-handed counterparts. These are the only superpotential terms necessary to recover standard model fermion masses and Higgs couplings. The MSSM possesses a $Z_2$ symmetry known as $R$-parity which can be represented by $R = (-1)^{F+L+3B}$ in terms of the particle's fermion, lepton, and baryon numbers.

## 2.2 Right-Handed Neutrinos and R-Parity Violation

One obvious extension of the MSSM consists of the inclusion of neutrino masses via a see-saw mechanism[2]. This is easily accomplished by the addition to the superpotential of

$$F_\nu = MNN + h_\nu H_1 LN \tag{2.3}$$

Neutrino masses of order ${h_\nu}^2 {v_1}^2/M$ will then be generated for the light (left-handed) neutrinos, where $v_1 = \langle H_1 \rangle$. It should be noted that the interactions induced by the superpotential $F_\nu$ do not violate R-parity as they only violate lepton number in units of two. They do not, however, constitute the most general set of N-field interactions allowed by gauge invariance. As well as $F_\nu$, one may also introduce the superpotential terms

$$F_N = \lambda H_1 H_2 N + kN^3 \tag{2.4}$$

The combination of the interactions in $F_\nu$ with *either* of the interactions in $F_N$ will result in violation of $R$-parity.

The inclusion of neutrino mass by the see-saw mechanism has many other benefits in addition to the generation of neutrino masses which can in principle aid in the solution to the solar neutrino problem and/or atmospheric neutrino deficit and/or cosmological hot dark matter (though not all simultaneously without the inclusion of a fourth sterile neutrino). Right-handed neutrino decay has been utilized[3] to generate a lepton asymmetry which in conjunction with non-perturbative electroweak interactions becomes a baryon asymmetry. In refs. [4, 5], this mechanism was extended to supersymmetric models as well. Another possibility[6] for the generation of a baryon asymmetry made use of flat directions in the scalar potential as in the Affleck-Dine mechanism [7]. In the latter, the superpotential $F = F_Y + F_N + F_\nu$ was required in order to induce a lepton number violating operator. For simplicity only one set (3 generations) of chiral superfields were added. Thus $R$-parity was explicitly violated. In that model, $R$-parity could have been preserved if the $N$-fields in $F_N$ were distinct and have a different $R$-parity assignment than that of the $N$'s in $F_\nu$.

There are numerous other ways in which one can imagine extending the MSSM. In what is often called the minimal-nonminimal superymmetric standard model (MNMSSM) a single additional gauge singlet chiral superfield, $N$ is added [8]. This extension is realized by simply adding to the superpotential the contribution from $F_N$ (2.4). The primary motivation for the inclusion of the Higgs singlet is the possibility that it offers for the dynamical generation of the Higgs mixing mass $\mu$. If the $N$ field is a field which acquires a vev determined by mass parameters of the order of the electroweak scale, then with a $NH_1H_2$ coupling of standard strength (say comparable to a gauge coupling) Higgs mixing of the requisite magnitude is induced. On the other hand, if the mass parameters in the $N$ sector are much larger, say of an intermediate scale, or perhaps of the GUT scale, as might naturally be expected to be in see-saw models, then if the $N$ has a nonzero vev one would naturally expect it to also be of this scale. In such a case one still might imagine inducing a weak scale mixing between the Higgs doublets, at the price of fine tuning the $NH_1H_2$ coupling to be small to give the hierarchical ratio between the electroweak scale and the $N$ mass scale. Though this small O($M_W$) mixing mass is technically feasible its smallness is part and parcel with the hierarchy problem. The cubic term is required in order to avoid an $N$-axion like field, in the absence of an explicit $\mu$ superpotential term mixing the two Higgs supermultiplets. In a detailed examination of this model [9], it was found that many of the standard Higgs mass relations are altered. If the MSSM Higgs mass relations are found to be experimentally not viable, this model becomes the simplest alternative.

From another point of view, the MNMSSM is of interest as it can easily produce a relatively light dark matter candidate [10]. In the MSSM, steadily improving accelerator limits, are pushing up the mass of the lightest supersymmetric particle (LSP), which due to the unbroken $R$-parity in the MSSM is stable. In the minimal model the LSP is generally expected to be a linear combination of the four neutral $R = -1$ fermions [11], the two gauginos, $\tilde{B}$ and $\tilde{W}$, and the Higgsinos $\tilde{H}_1$ and $\tilde{H}_2$. With regards to a dark matter candidate, the best choice in the MSSM appears to be the bino whose mass is typically between 40 GeV and $\sim$ 300 GeV for cosmologically interesting parameters [12]. In the non-minimal model it is quite feasible [10, 13] to have a light LSP (10 - 50 GeV), which is a state which has a strong admixture of the fermionic component of $N$. Though, cosmologically, a very massive LSP is just as good as a light one (light still referring to O(GeV)), from the point of view of experimental detection, the lighter one is better [14].

In this chapter we derive the consequences of the $R$-parity violation of the full superpotential. $R$-parity violation in the quark sector is usually avoided in order to insure a relatively stable proton. In the Higgs-lepton sector, there are many constraints on $R$-parity violation as well. In the case we consider here, $R$-parity is violated only in the heavy $N$-field sector. Nevertheless, this $R$-parity violation shows up in the low energy sector, most notably in the destabilization of the LSP. We derive constraints on the neutrino mass parameters as a consequence of the constraints on late-decaying LSP's.

## 2.3 Slepton-Higgs Mixing and LSP Decay

In addition to the destabilization of the LSP to which we will turn below, there are other possible low-energy signatures of R-parity violation in the high energy N-field sector. If supersymmetry were exact, then even the combined presence of the $F_\nu$ and $F_N$ superpotential terms would not induce (super)renormalizable lepton number violating superpotential terms involving only the light superfields of the theory, due to the nonrenormalization theorems for the superpotential. After supersymmetry breaking the nonrenormalization theorems no longer hold exactly, and lepton number (and hence R-parity) violating effective interactions will be induced in an amount governed by the scale of supersymmetry breaking. This will result in low energy R-parity violating interactions involving standard model superfields of the form of both induced effective superpotential terms such as

$$F_{RX} = m_X H_1 L + \lambda_X L L e^c \tag{2.5}$$

as well as soft supersymmetry breaking lepton number violating terms. As we will show below, the nature of the R-parity violation emerging at low energies depends decisively on whether it arises from the $NH_1H_2$ term or the $NNN$ term in the superpotential. Supersymmetry breaking R-parity violating slepton-Higgs soft mixing terms will arise in a way that depends crucially on the form of the R-parity violation in the $N$-field sector, and in some cases may not be suppressed for large $N$-field masses. By appropriate change of basis we may diagonalize the Higgs-lepton mass mixing and parametrize our lepton number violating effects by $\lambda_X$. Soft supersymmetry and R-parity violating mass terms which are induced in the low energy theory may also be rotated into effective trilinear interactions by rediagonalization of mass terms in the slepton-Higgs state space. Lepton number violating renormalizable interactions of this type are constrained by laboratory limits on lepton flavour violation, and neutrinoless double beta decay [15]. As we have analyzed previously, even stronger limits are imposed on interactions of this type by the persistence of a baryon asymmetry in the early universe, assuming that it is not produced at or after the electroweak phase transition [16, 17]. The danger here is that the lepton number violation implied by the new interaction could attain thermal equilibrium at the same time as baryon and lepton number violating (but B-L conserving) nonperturbative electroweak interaction effects to simultaneously equilibrate both the baryon and lepton number of the universe to zero. If these limits pertain, they would imply that $\lambda_X < 7 \times 10^{-7}$ [16]. These limits may be evaded, and indeed a baryon asymmetry may be generated from a lepton asymmetry, provided one of the generations of lepton flavours has its lepton number violating interaction in equilibrium, while another does not [18]. As we have mentioned above, the combination of the $NH_1L$ superpotential term with

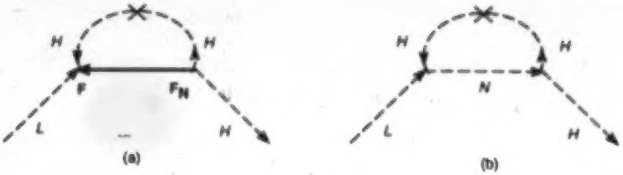

Figure 2.1: Diagrams for slepton-Higgs mixing induced by an $NH_1H_2$ superpotential term.

either the $NH_1H_2$ or $NNN$ superpotential interactions breaks R-parity and hence will destabilize the lightest neutralino mass eigenstate. The nature of the low energy R-parity violation induced, and the rate of the resulting LSP decay will depend on which of these latter terms is responsible.

It is well known that in supersymmetric theories with soft supersymmetry breaking, renormalization group running of soft scalar mass terms [19] may induce, in the low energy sector, soft scalar mass mixings which result from interactions with (super)fields from the high energy sector of the theory [20]. This results, for example, in lepton flavour violating contributions to slepton mass mixings induced by GUT [20], or by heavy singlet see-saw [21] superpotential couplings. So the question that presents itself is whether the introduction of R-parity violation in the heavy N-field sector will be fed down by renormalization group mixing to induce soft R-parity violating slepton-Higgs mass mixings in the low energy theory.

One can see from the diagrams in Fig. 2.1 how such slepton-Higgs soft mixings will be induced by the simultaneous presence of the $h_\nu H_1 L N$ and $\lambda H_1 H_2 N$ superpotential terms. For loop momenta below $M_N$ the diagrams will cancel, but for loop momenta greater than $M_N$ the diagram of Fig. 2.1(b) will be suppressed by the $N$ propagator. The surviving contribution from Fig. 2.1(a) will be proportional to the product of the couplings, the soft supersymmetry-breaking scalar mass squared, and the logarithm of the ratio of the cutoff scale $\Lambda$ (say of order the Planck scale) to the $N$-field mass (below which the cancellation between the diagrams is reinstated). This logarithm of the cutoff is the term resummed in the renormalization group mixing, when running the renormalization mass scale from the cutoff scale down to the $N$-field mass scale. The resulting induced soft mass mixing squared is then

$$\Delta M^2_{LH} \propto \lambda^\dagger h_\nu m_0^2 log(\Lambda/M_N) \tag{2.6}$$

and does not decouple for large $M_N$. So in this case the R-parity violation in the $N$-field sector is fed down by the renormalization group to soft scalar Higgs-slepton mixings in the low energy sector, with the resulting implications for lepton number and flavour violation, destabilization of the LSP (see below), and lepto/baryogenesis.

If, on the other hand, the R-parity violation enters the $N$-field sector via the $kNNN$ term, then renormalization group mixing will not induce soft slepton-Higgs mixing. In this case, the one loop diagrams with the R-parity violating trilinear F-term and soft supersymmetry breaking mass insertion are shown in Fig. 1.2. As the $N$-field F-component in Fig. 2.2(a) and the $N$-field scalar in Fig. 2.2(b) carry zero momentum, these diagrams exactly cancel, and there is no induced mixing [1]. Thus the mechanism by which R-parity violation in the $N$-field sector comes to destabilize the LSP depends crucially on the manner in which that R-parity violation arises.

First let us consider LSP decays induced by the $NH_1H_2$ superpotential term. With this coupling the renormalization group running generates slepton-Higgs mixing of order the susy breaking scale times the couplings, up to factors of small logs. The LSP can now decay from its $\tilde{H}_2^o$ component. This component has an F-term coupling to a lepton-slepton pair via the $h_e H_2 L e^c$ superpotential coupling. With the slepton-Higgs mixing we have the physical decay to a lepton-Higgs final state. If (in the absence of mixing with the N-field) we would write the LSP as an admixture

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[1] This cancellation is the same one that allows the decoupling of heavy superfields from tree level interactions of light superfields which are coupled to them, where the heavy field F-term contribution to the light field potential is cancelled by the heavy scalar exchange contribution.

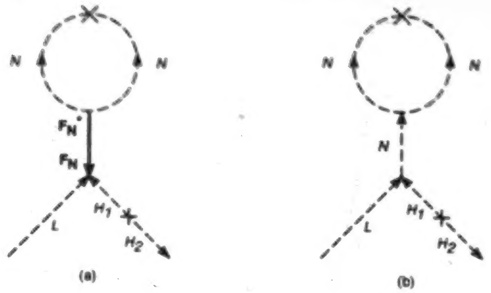

Figure 2.2: Diagrams giving cancelling contributions to the slepton-Higgs mixing, which arise from $NNN$ superpotential term.

$$\tilde{\chi}_o = \alpha\tilde{B}^o + \beta\tilde{W}^o + \gamma\tilde{H}_1^o + \delta\tilde{H}_2^o \tag{2.7}$$

then the decay of the LSP via its $H_2$ component will then occur at a rate

$$\Gamma \simeq 4\delta^2\lambda^2 h_\nu^2 \Gamma_o \tag{2.8}$$

where

$$\Gamma_o \simeq \frac{m_{\tilde{\chi}_o}}{16\pi}(1 - \frac{m_o^2}{m_{\tilde{\chi}_o}^2}) \tag{2.9}$$

where $m_o$ is the mass of the final state Higgs scalar, and we have assumed that the soft scalar masses, and mass mixings, are of order the Higgs mass.

There will also be a contribution to the neutralino decay width from decay of its $H_1$ component mediated by N-fermion exchange, with a contribution

$$\Gamma \simeq \gamma^2\frac{\lambda^2 h_\nu^2 v^2 \sin^2\beta}{M_N^2}\Gamma_o \tag{2.10}$$

it is suppressed by the $N$ mass, and will not, in general, be significant.

Similarly, decays of the LSP may be induced by the $NNN$ superpotential term. As we have discussed above, in this case there are no soft lepton number violating $\tilde{L}H$

mass mixing terms induced by renormalization group mixing. Now the dominant decay modes are those arising from the diagrams of Fig. 2.3. We note that Fig. 2.3(a) is an induced D-term and contains a loop which is also a D-term. This ensures a non-zero decay rate for the neutralino even when supersymmetry is unbroken, unlike the case for F-terms. Because D-terms do not obey non-renormalization theorems, they can be radiatively induced even in the limit of unbroken supersymmetry; hence in general they appear without suppression factors associated with the scale of supersymmetry breaking. We also note that the induced D-term in Fig. 2.3(a) (and its associated component diagrams) is a dimension six term [22]. The component diagrams relevant to neutralino decay are shown in Figs. 2.3(b) to 2.3(e). The processes of Fig. 2.3 dominate over decays induced by tree-level diagrams for large $M_N$, as the latter are suppressed by eight powers of $M_N$ in rate, whereas the loop induced decays are only suppressed by four powers.

Computing the diagrams of Fig. 2.3(b) and Fig. 2.3(c) one finds that they result in a decay rate that is approximately

$$\Gamma \sim \gamma^2 \frac{k^2 h_\nu{}^6}{16\pi(2\pi)^8} \mu^2 \frac{v_1{}^2 m_{\tilde{\chi}_0}}{M_N{}^4} \tag{2.11}$$

whereas the final two diagrams of Fig. 2.3 result in a decay rate for the LSP that is approximately

$$\Gamma \sim \gamma^2 \frac{k^2 h_\nu{}^6}{16\pi(2\pi)^8} \frac{v_1{}^2 m^3_{\tilde{\chi}_0}}{M_N{}^4} \tag{2.12}$$

We expect that the Higgsino mass should be at least of the order of the doublet mixing term, and in certain circumstances the doublet mixing term might be substantially smaller; so below we will use the latter of these rates for numerical estimates.

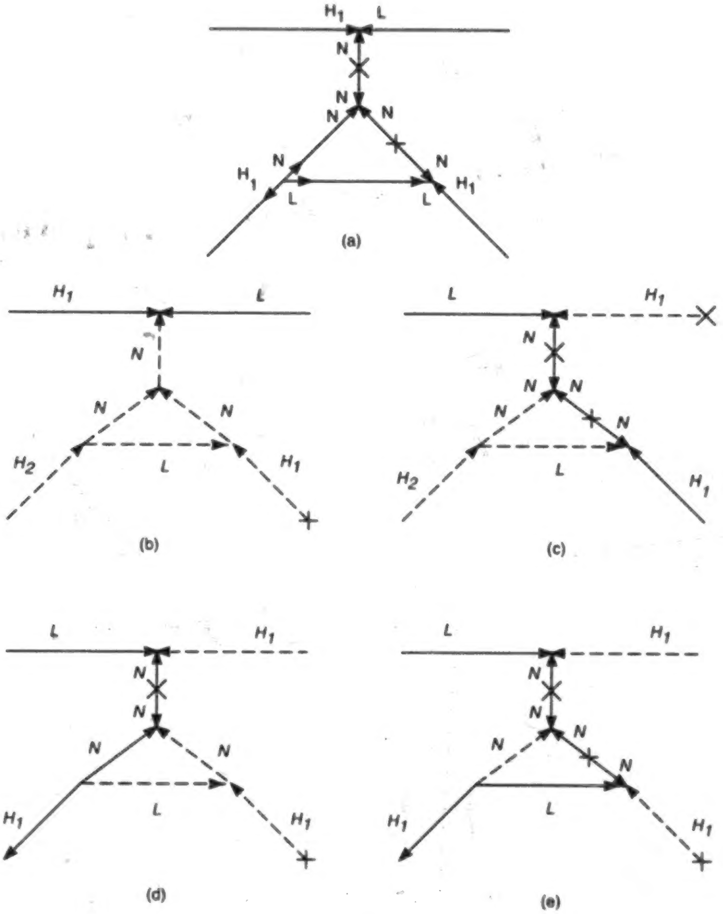
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Figure 2.3: Neutralino decay diagrams by an $NNN$ superpotential term. Fig. 2.3(a) is the superfield diagram whose dominant associated component field diagrams include those shown in Figs. 2.3(b) through 2.3(e).

### 2.3.1 Comparison with Cosmological Limits

Almost without exception, the LSP decays we are considering are effectively entropy producing decays, i.e. they will produce high energy photons. Photon producing decays are known to be highly constrained from both astrophyical and cosmological observations (see eg. ref. [23] for a recent compilation of such limits). These limits generally place constraints in the density-lifetime plane of the decaying particle. We will assume that the LSP, in the absence of its decay, is the dominant form of dark matter and therefore assume that its cosmological density is such that $\Omega_\chi \approx 1$, where $\Omega = \rho/\rho_c$ is the cosmological density parameter. At this density, one finds that the LSP lifetime is constrained so that either $\tau_\chi \lesssim 10^4$s to avoid affecting the light element abundances produced during big bang nucleosynthesis, or the LSP must be effectively stable with a lifetime $\tau_\chi \gtrsim 10^{24}$s. Astrophysical limits on other R- parity violating interactions were considered in [24].

The decay rates that we derived are clearly dependent on a number of model parameters. In order to get a feeling for the limits imposed by the cosmological constraints we make a few more assumptions. We assume that the LSP is primarily a gaugino (a bino) with mass $m_\chi \approx 150$ GeV. For $|\mu| \sim 1 - 10$ TeV, $\gamma \sim 2 \times 10^{-3} - 2 \times 10^{-2}$ and $\delta \sim 4 \times 10^{-3} - 4 \times 10^{-2}$ and for large $\tan\beta$, $\sin\beta \approx 1$. We can then write (for the decays based on the $H_1 H_2 N$ superpotential term)

$$\tau_\chi \simeq 10^{-25} \left( \frac{150 GeV}{\delta^2 \lambda^2 h_\nu^2 m_\chi} \right) s. \tag{2.13}$$

This unsuppressed (by factors of the $N$ mass) decay is, from the cosmological viewpoint, effectively instantaneous, and not subject to constraint, save that of the absence of LSP dark matter, and those of maintenance (or regeneration) of the baryon asymmetry at temperatures above the electroweak scale.

For LSP decay induced by the $kN^3$ superpotential term, from the decay width estimate given above we deduce an LSP lifetime of order

$$\tau_\chi \simeq 4 \times 10^{20} h_\nu{}^{-6} k^{-2} \gamma^{-2} \left(\frac{M_N}{10^{12}\mathrm{GeV}}\right)^4 \left(\frac{150\mathrm{GeV}}{m_\chi}\right)^3 \mathrm{s} \tag{2.14}$$

which translates into the limits

$$M_N \leq 5 \times 10^6 h_\nu{}^{3/2} k^{1/2}\ \mathrm{GeV} \tag{2.15}$$

or

$$M_N \geq 5 \times 10^{11} h_\nu{}^{3/2} k^{1/2}\ \mathrm{GeV} \tag{2.16}$$

## 2.4 Conclusion

These limits show therefore that even if $R$-parity violation is inserted in the singlet sector, destabilization of the LSP can indeed occur and $R$-parity violation of this type is strongly constrained. It is especially interesting that cosmological arguments provide such strong constraints, probing possible see-saw sources of R-parity violation to far higher mass scales than could be directly accessed by laboratory experiment; this provides yet another example of the power of cosmological considerations to provide us with new information about the fundamental interactions of nature.

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# Chapter 3

# Cosmological Reheating and Self-Interacting Final State Bosons

## 3.1 Introduction

Inflationary cosmology [1], provides solutions to the flatness, isotropy and stable relic problems of the standard hot big bang. In these models the universe experiences a period of superluminal expansion during which its energy density is dominated by the potential energy of a scalar field (inflaton). Inflation ends when the inflaton enters the oscillatory regime during which we have a matter-dominated FRW-universe, after which the inflaton decays to relativistic particles (reheating). Reheating represents the crucial transition from the epoch of scalar-field dominated dynamics to a hot FRW universe. After reheating, the universe becomes radiation-dominated and its evolution is just that of the standard hot big bang .

In the standard picture of reheating [1, 2] the effective decay of the inflaton occurs when $\Gamma \simeq H$, where $\Gamma$ is the one particle decay rate, and the reheat temperature of the universe is $T_R \sim 0.1(\Gamma)^{\frac{1}{2}}$ (from now on $M_{pl} = 1$ and all dimensionful quantities are expressed in these units unless otherwise indicated). In this picture, which we will refer to as the linear regime, perturbation theory is taken to be valid and the occupation number of the final state bosons is assumed to be smaller than one (for fermions this is assured because of Pauli blocking). It has recently been recognized within different approaches [3]-[31] that this picture is incomplete, and nonlinear effects can change it essentially, leading by parametric amplification to an enhanced decay of the inflaton (for a recent review see, e.g. [3]). There are two different possible regimes of such a parametrically amplified decay. In the first the decay occurs over many oscillation times of the inflaton, but experiences parametric amplification with moderately large occupation numbers for the modes of the decay product field; this "narrow-band resonance" case is amenable to analytic calculation, and we will be able to analyze the modifications to parametric amplification due to final state-self interactions of the decay products quantitatively. In the second case of "broad-band resonance" the amplification of the decay is so strong that there is explosive decay of the inflaton field on a time-scale not hierarchically longer than the oscillation time, with large occupation numbers for the modes of the decay product field; this scenario is more difficult to analyze quantitatively, but the physical mechanisms which we discuss are of sufficient generality that we expect that they alter the decay dynamics in this case also.

The consequences of a parametrically amplified decay could be significant, and pose major difficulties in the construction of viable inflationary models. If the decay is very fast, the final state particle modes have, in general, very large occupation numbers and are far from thermal equilibrium; these fluctuations have certain effects similar to very high temperature thermal corrections [9, 12], and after thermalization may themselves lead to high reheat temperatures, with thermal energy densities of order the inflationary energy density. While on the one hand GUT symmetry restoration due to these non-thermal configurations revives GUT scenarios for baryogenesis [23, 16, 17] , on the other hand it reintroduces the problem of heavy topological defects whose solution was one of the initial motivations for inflation.

Furthermore, realistic models seeking to implement inflation need to be able to stabilize the required flat potential against radiative corrections. The only presently known method to achieve this in the presence of gauge, scalar potential, Yukawa, and gravitational interactions is by using (approximate) supersymmetry to enforce the appropriate non-renormalization. Parametrically amplified decay of the inflaton, and the resulting efficient reheat, poses a mortal danger to realistic, supersymmetric, inflationary models. In supersymmetric theories the reheat temperature is constrained [32, 33, 34] by the need to avoid thermal overproduction of gravitinos. This leads to an upper bound on the reheat temperature of order $10^8 - 10^9$ GeV, which is orders of magnitude below what would be achieved by efficient reheat from amplified inflaton decay. One obvious way to avoid this disaster would be to insure that the inflaton is sufficiently weakly coupled to its bosonic decay products that the decay never experiences parametric amplification. This is typically the case for inflatons in a hidden sector which have only gravitational strength couplings to their (observable sector) decay products. On the other hand, in many models one introduces direct superpotential couplings of the inflaton to the chiral scalars into which it decays, and we wish to consider the nature of the reheat dynamics in such models, and whether they are ruled out by constraints on reheating.

In particular, we will examine the effects that the final state self-interactions of the decay products have on the parametric amplification of the inflaton decay. Because of the large occupation numbers for the modes of the produced decay bosons, we expect that the presence of self-interactions of these bosons will result in large effective masses being induced for these modes. If the bosons are thermalized these may be interpreted as thermal plasma masses from self-interaction; more generally they will occur as induced effective plasma mass terms in the mode equations for the decay field. As the mode occupation numbers increase, so do these induced masses, until they equal the mass of the inflaton, cutting off the decay. Decay resumes with the thermalization of the decay products, and their dilution and redshift by cosmic expansion, such that their induced masses dip below the inflaton mass; the system thus proceeds in a quasi-stationary process of decay and dilution such that the induced mass of the decay products is always of order the inflaton mass. This regulates the parametric amplification of the decay, preventing abrupt and efficient reheat. In our discussion we will analyze the effects of final state self-interactions for inflaton decays that would otherwise be in the regime of narrow-band parametric amplification. However, because the dominant effect is a kinematical cutoff of the decay, due to the self-induced plasma mass of the final state decay products arising from their strong self-interaction, we expect similar effects in the case of broad-band resonance. Indeed, recent studies of the broad-band decay regime [7, 10, 14] indicate that the explosive decay from the sequence of higher resonance bands can only effectively produce particles whose mass does not exceed that of the inflaton by more than an order of magnitude. So in this case too there will be a kinematical cutoff due to the final state self-interaction induced plasma mass of the decay products; although it will now be regulated to be not more than of order ten times the inflaton mass the qualitative effects should be otherwise similar. Applications of our present analysis to realistic classes of supersymmetric inflationary models will be considered elsewhere [31].

## 3.2 Parametric Amplification and Final State Self-Interactions

As a basis for our subsequent arguments we will consider a chaotic inflation model with the following potential, whose features we take to resemble the generic features of scalar potentials which arise in supersymmetric theories:

$$V = \frac{1}{2}m^2\phi^2 + \sigma\phi\chi^2 + h^2\phi^2\chi^2 + g^2\chi^4 \tag{3.1}$$

where for schematic simplicity the inflaton $\phi$ and the matter scalar $\chi$ are taken to be real scalar fields and the self-coupling of the $\chi$ field is considered to be of moderate strength $10^{-1} < g^2 < 1$. In supersymmetric theories where the decay scalars are standard model chiral scalars which are gauge non-singlets the quartic potential terms in $\chi$ arise as D-terms and the coupling is of gauge coupling strength $g^2$. For inflaton-scalar couplings arising from superpotentials in supersymmetric theories one has $\sigma = 2hm$, and the cubic and quartic couplings of the inflaton to $\chi$ are related to each other. In general the superpotential couplings $h$ may be, and in viable supersymmetric inflationary models are usually chosen to be, much smaller than gauge couplings, $h \ll g$. The inflaton mass $m$ must be bounded by $m \lesssim 10^{-6}$ in order to be consistent with COBE data on microwave background fluctuations [35].

First let us review the effects of parametric amplification on inflaton decay, ignoring self-interaction of the decay products. The nonlinear effects that lead to amplified decay act in two different regimes:

[1] $\frac{m^4}{\sigma^2} \lesssim \phi \lesssim \frac{m^2}{\sigma}$ for the cubic coupling and $\frac{m^4}{h^4} \lesssim \phi^3 \lesssim \frac{m^3}{h^3}$ for the quartic coupling. This is the narrow-band resonance case which can be analyzed perturbatively, and the dominant effect is the large occupation number for $\chi$'s. This case has been considered in [7, 8, 10, 11, 18] , where it is shown that parametric amplification occurs and there are narrow-band resonances for $\chi$ production at $k = \frac{m}{2}$ and $k = m$ for the cubic and quartic couplings respectively. This is the case for which we will make quantitative estimates of the effect of inclusion of final state self-interaction of the decay products.

[2] $\phi \gtrsim \frac{m^2}{\sigma}$ for the cubic coupling and $\phi \gtrsim \frac{m}{h}$ for the quartic coupling. Here perturbation theory is not valid and problem is highly nonlinear. In this case there is broad-band resonance for a large domain of momenta [7, 10] that leads to an explosive decay of the inflaton. As noted above, because of the essentially kinematic nature of the cutoff we expect the final-state self-interaction effects in this case to be qualitatively similar to those in the narrow band case.

Note that if the cubic and quartic couplings of the inflaton to the decay scalar are of the form arising from a superpotential coupling [$\sigma \simeq 2hm$], then the conditions for parametric amplification are more general for the cubic coupling and it will dominate the decay. Another important observation is that although most inflatons may decay during a stage of amplified decay this does not lead to the decay of the entire energy density of the inflaton. In the case of narrow-band resonance the decay stops no later than the time when $\phi$ has the minimum value in the abovementioned range ($\frac{m^4}{\sigma^2}$ for the cubic coupling and $(\frac{m}{h})^{\frac{4}{3}}$ for the quartic coupling) [7, 8], while in the case of broad-band resonance it stops at the time of the transition from broad-band to narrow-band resonance when the decay becomes out of equilibrium [7, 10, 25]. After that, the remaining energy density of the inflaton is redshifted as $a^{-3}$ where energy density of relativistic $\chi$'s is redshifted as $a^{-4}$. The decay of the inflaton will then be completed as in the usual picture, and if the energy density of the inflaton dominates at that time there will be significant dilution of relic densities from the first stage of reheat [25, 16]. In supersymmetric theories this dilution is not, generally, sufficient by itself to solve the problem of overproduction of gravitinos. However, combined with the effect of final state self-interaction, which we consider below, it can successfully resolve the gravitino problem in many models.

Now let us consider generally the changes to the parametric amplification reheat dynamics that arise from the self-interactions of the final state decay products. Since by assumption the inflaton decays to observable sector standard model (s)particles the final state bosons carry gauge quantum numbers. For these fields self-interactions with couplings as strong as the gauge coupling arise at tree-level from D-terms in supersymmetric models [1], and at the one-loop level in the non-supersymmetric case. In addition, there are couplings of non-singlet scalars to the gauge fields as well as possible large superpotential couplings. The decay of the inflaton produces quantum fluctuations along the direction of final state particles in field space and drives them to large field values where the effect of self-coupling becomes important. These large field values induce a self-mass for the decay products that subsequently slows down the decay and tends to shut it off .

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[1] There are, of course, directions in scalar field space which are both D-flat and F-flat in the supersymmetric standard model; in realistic no-scale supergravity models they will in general already have developed Planck scale vevs during inflation [36]. If the decay coupling of the inflaton happened to align along one of these directions then final state interactions of the type we consider would not affect this particular decay mode. More generally the decay products will themselves, in turn, have decay modes along these directions. We consider these issues elsewhere [31].

During the oscillatory regime $\phi \cong \phi_0 \cos(mt)$, and immediately after the end of inflation $\phi_0 \sim 10^{-1}$. Subsequently $\phi_0$ decreases with time because of decay and Hubble expansion. For $\phi_0 \lesssim \frac{m}{h}$ the inflaton decays predominantly via the cubic term. As was discussed in [7, 8, 10], in the range $(\frac{m}{h})^2 \lesssim \phi_0 \lesssim \frac{m}{h}$ parametric amplification occurs and there is a narrow-band resonance for $\chi$ production at $k = \frac{m}{2}$. In this range for the inflaton field we can use the Feynman diagrams for one-particle decay, but the occupation number of final state particles is non-trivial and must be taken into account in the calculations. According to [7, 8, 10] parametric amplification will effectively convert most of the energy density of the inflaton into radiation once it is in the afore-mentioned range. Assuming rapid subsequent thermalization this leads to a reheat temperature $T_R \sim mh^{-\frac{1}{2}}$ that is much higher than that of the usual picture $\sim 0.1 m^{\frac{1}{2}} h$ for reasonable values of $h$. The quartic self-coupling of $\chi$ will induce a finite temperature correction to the mass-squared of $\chi$, if they are thermalized, of order $g^2 {T_R}^2 \sim g^2 m^2 h^{-1}$ at this time which is much larger than $m^2$ (as we mentioned above the non-thermal corrections that exist before thermalization are even larger). However at an earlier time $t_d$ when the thermal (or non-thermal [2]) correction is of order $\frac{m^2}{4}$ the one-particle decay becomes kinematically forbidden (note that the thermal correction to mass-squared of $\phi$ is of order $h^2 {T_R}^2$ which is normally smaller than $m^2$ for $h$ a typical Yukawa type coupling).

The Hubble expansion that subsequently occurs redshifts the correction to the mass-squared of $\chi$ as $a^{-2}$ and this causes further decay. As long as $t_d < H^{-1}$ these successive steps of expansion, decay and (perhaps) thermalization continue. Eventually the decay is not effective enough to compensate for expansion and there is a delay before the remaining energy density of the inflaton is converted into relativistic particles as in the usual picture, and the decay of the inflaton is completed.

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[2] Whether the thermal or non-thermal correction should be considered depends on how rapid the thermalization rate is. Details will be discussed shortly.

## 3.3 Mode Analysis of Mathieu Equation

Now let us perform a detailed mode by mode analysis of the effect of quartic self-coupling of final state particles. Consider the potential

$$V = \frac{1}{2}m^2\phi^2 + 2hm\phi\chi^2 \tag{3.2}$$

where $\phi$ and $\chi$ are both real scalars. By mode expansion of $\chi$ we derive the following equation for each mode

$$\ddot{\chi}_k + 3\frac{\dot{a}}{a}\dot{\chi}_k + (k^2 + 4hm\phi_0 \cos mt)\chi_k = 0 \tag{3.3}$$

where a dot denotes differentiation with respect to time, and in this equation for the modes associated with comoving wavenumbers, we are using the physical wavenumber $k$, where $ak = k_{comoving}$ with $a$ the scale factor . For the moment we will ignore the effect of Hubble expansion, since it generally occurs over a longer time scale than that of the effects we consider. We will consider issues of thermalization and Hubble expansion below [for other treatments of the effect of Hubble expansion on the parametric resonance see [7, 8, 11]]. By choosing $z = \frac{m}{2}t$, in the absence of final state self-interactions we derive a Mathieu equation for the modes of the $\chi$ field,

$$\chi_k'' + (\frac{k^2}{(\frac{m}{2})^2} + \frac{4hm\phi_0}{(\frac{m}{2})^2}\cos 2z)\chi_k = 0 \tag{3.4}$$

where a prime denotes differentiation with respect to $z$. In the case of narrow-band resonance (in which we perform our calculations) the Mathieu equation has resonance solutions in the first instability band $(\frac{m}{2})^2 - 4hm\phi_0 < k^2 < (\frac{m}{2})^2 + 4hm\phi_0$. Modes in this band grow exponentially in time, which one interprets as particle production. The (slow) Hubble expansion will eventually drive the modes out of the instability band, but they spend enough time there to reach a substantial occupation number [18]. This time $t_b$ can be approximately calculated from $\delta k \sim kHt_b$ where $k = \frac{m}{2}$ and $\delta k = 8h\phi_0$ is the width of the first instability band, which gives $t_b \sim \frac{16h}{m^2}$.

The quartic self-coupling $g^2\chi^4$ will induce an effective mass-squared $m^2{}_{eff}$ for the $\chi$ field and the Mathieu equation with the addition of this self-coupling will be modified to become

$$\chi_k'' + (\frac{k^2 + m^2{}_{eff}}{(\frac{m}{2})^2} + \frac{4hm\phi_0}{(\frac{m}{2})^2}\cos 2z)\chi_k = 0 \tag{3.5}$$

In a background of isotropically distributed $\chi$'s over a narrow band of momenta (which is the case in $\phi \rightarrow \chi\chi$ decay before thermal distribution is achieved) $m^2{}_{eff} \sim g^2\frac{n_\chi}{E_\chi}$ to the leading order, with $n_\chi$ the number density of $\chi$'s and $E_\chi$ their energy [7, 10, 12, 13, 14]. After thermalization, in a thermal background of temperature $T$, we will have the standard result $m^2{}_{eff} \sim g^2T^2$.

At $t = 0$, when $\phi$ starts oscillating, $m^2{}_{eff} = 0$ and there is resonance in the band $(\frac{m}{2})^2 - 4hm\phi_0 < k^2 < (\frac{m}{2})^2 + 4hm\phi_0$. With particle production in this band, $m^2{}_{eff}$ increases, and resonance in this band stops when $m^2{}_{eff} = 8hm\phi_0$, after which the number density of particles produced in this band remains unchanged. There will be, however, resonance in the band $(\frac{m}{2})^2 - 12hm\phi_0 < k^2 < (\frac{m}{2})^2 - 4hm\phi_0$ that also stops when $m^2{}_{eff}$ increases by a further amount of $8hm\phi_0$. This incremental change of $m^2{}_{eff}$ and a smooth transition from one resonance band to the next one continues until $m^2{}_{eff} = (\frac{m}{2})^2$, after which there is no resonance solution for physical states, i.e. states with $k^2 > 0$. If we take $E_\chi \simeq (\frac{m}{2})$ for particles in all bands, a change of $8hm\phi_0$ in $m^2{}_{eff}$ correponds to an increase in number density of $\chi$'s by $\delta n_\chi \sim \frac{4hm^2\phi_0}{g^2}$. For the first band $(\frac{m}{2})^2 - 4hm\phi_0 < k^2 < (\frac{m}{2})^2 + 4hm\phi_0$, and for $h\phi_0 \ll m$ the occupation number $f_{\frac{m}{2}}$ required to shift the resonance to the next band is calculated to be

$$\delta n_\chi \sim \frac{1}{(2\pi)^3} f_{\frac{m}{2}} \times 4\pi(\frac{m}{2})^2 \times 8h\phi_0 \sim \frac{4h\phi_0 m^2}{g^2} \Rightarrow f_{\frac{m}{2}} \sim \frac{4\pi^2}{g^2} \tag{3.6}$$

It has been shown [18] that in the small amplitude limit the analytic result of the decay rate in the $n$-th instability band can also be derived from a physical process, $n$ particles with zero momentum that comprise the classical homogeneous inflaton field annihilating into two final state bosons. In particular, for the first instability band and for $f_k \ll 1$ we can use the one-particle decay rate from the standard perturbation theory $\Gamma = \frac{h^2 m}{8\pi}$. Narrow-band parametric amplification has been shown to be indeed an induced process [18] which means for $f_k \gtrsim 1$ the enhancement of decay rate by the factor $(1 + f_k)$ must be taken into account in the particle calculation. The time $t_1$ which is needed to reach $f_{\frac{m}{2}} = 1$ can be approximately calculated as [3]

$$\frac{h^2 m}{8\pi} t_1 m {\phi_0}^2 \sim \frac{\frac{2hm^2\phi_0}{g^2}}{\frac{4\pi^2}{g^2}} \Rightarrow t_1 \sim \frac{4}{\pi h \phi_0}, \tag{3.7}$$

and considering the enhancement factor the time $\delta t$ which is needed to reach $f_{\frac{m}{2}} = \frac{4\pi^2}{g^2}$ is

$$\delta t \sim \frac{4}{\pi h \phi_0} \ln \frac{4\pi^2}{g^2}. \tag{3.8}$$

This is the time for the resonance in the band $(\frac{m}{2})^2 - 4hm\phi_0 < k^2 < (\frac{m}{2})^2 + 4hm\phi_0$ to stop. For the next bands $k^2$ is smaller and this means that less phase space volume is available for decay products or, equivalently, the occupation number for those bands is larger. Therefore, more time will be needed for production of $\delta n_\chi \sim \frac{4hm^2\phi_0}{g^2}$ in bands with smaller $k^2$, but not greatly so, as the larger occupation numbers are obtained rapidly due to the large coherent final state enhancement. A reasonable lower estimate for the decay time $t_d$ to effectively achieve ${m^2}_{eff} \sim \frac{m^2}{4}$ is

$$t_d \sim \frac{\frac{m^2}{4}}{8hm\phi_0} \delta t \sim \frac{m}{8\pi h^2 {\phi_0}^2} \ln \frac{4\pi^2}{g^2} \tag{3.9}$$

In order to have physically realistic estimates we take $g^2 \cong 10^{-1}$ in our calculations. This leads to

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[3] Note that $e^{-\Gamma t_1} \simeq 1 - \Gamma t_1$ for $\Gamma t_1 \ll 1$, which is valid here.

$$\delta t \sim \frac{8}{h\phi_0} \quad , \quad t_d \sim \frac{m}{4h^2{\phi_0}^2}. \tag{3.10}$$

So resonance at each band of width $8hm\phi_0$, ends in a time $\sim \frac{8}{h\phi_0}$ and in an approximate time of $\frac{m}{4h^2{\phi_0}^2}$ decay effectively stops. We notice that $t_d < H^{-1}$ for $\phi_0 \gtrsim (\frac{m}{h})^2$.

In this analysis the effect of self-coupling of $\chi$ on the solutions of the Mathieu equation was considered only to the first order, which is reasonable for the case of narrow-band resonance $h\phi_0 \lesssim m$. The key assumption in our treatment of the Mathieu equation in the presence of the nonlinear term was adiabaticity, i.e. that we can use the instantaneous value of ${m^2}_{eff}$, which is also legitimate since it changes over a time $\delta t \sim \frac{8}{h\phi_0}$ which is greater than $m^{-1}$, the period of oscillations of $\phi$, again because we are in the narrow-band resonance regime $h\phi_0 \lesssim m$.

Our results show major differences from the simple parametric amplification case. The effect of large occupation number for final state particles is not that dramatic here because there is a whole range of resonance bands instead of a single one and the effect of the self-interaction of the produced particles drives modes out of the resonance bands much faster than the Hubble expansion. Consequently the leading effect that influences the decay is the self-interaction of the decay products, which stops it very early.

## 3.4 Thermalization and Hubble Expansion

So far we have not considered thermalization of decay products and the Hubble expansion. The temperature of the thermal bath after thermalization of $\chi$'s is calculated from [4]

$$\frac{\pi^2}{30}(g_B + \frac{7}{8}g_F)T^4 = \rho_\chi = n_\chi E_\chi \sim \frac{m^4}{16g^2} \tag{3.11}$$

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[4] In general the number density of particles is not the same before and after thermalization but the energy density is conserved.

where $g_B$ , $g_F$ are the number of bosonic and fermionic degrees of freedom, respectively. We take the number of degrees of freedom to be the one in the minimal supersymmetric standard model ($g_B = g_F = 74$) which leads to a temperature $T \simeq g^{-\frac{1}{2}}\frac{m}{5}$. For this temperature, however, the correction to mass-squared of $\chi$ is

$$m^2{}_{eff} \sim g^2 T^2 \sim g\frac{m^2}{25} \tag{3.12}$$

which is much less than $(\frac{m}{2})^2$ for $g^2 \cong 10^{-1}$. This is just what we expected because $m^2{}_{eff} \sim g^2 \frac{n_\chi}{E_\chi}$ will be smaller after thermalization when the number density decreases and the mean energy of particles increases. Therefore if thermalization is effective, it will lower $m^2{}_{eff}$ significantly which leads to further decay and thermalization. This sequence of decay and thermalization stops when $g^2 T^2 \sim \frac{m^2}{4}$ that is at a temperature $T \sim m$ for $g^2 \cong 10^{-1}$ if the sequence is completed within a Hubble time.

Considering thermalization of decay products and the Hubble expansion, there are different possibilities depending on the relation among different time scales involved in the problem, $t_{osc} \sim m^{-1}$, $\delta t \sim \frac{8}{h\phi_0}$, $t_d \sim \frac{m}{4h^2\phi_0{}^2}$, $t_{th} \sim \frac{32\pi}{\alpha m}$, [5] and $t_H \sim \sqrt{\frac{3}{8\pi}}\frac{1}{m\phi_0}$. The most important thing is that $t_{osc}$ is considerably smaller than all other time scales as long as we are in the narrow band regime $\phi_0 \lesssim \frac{m}{h}$. Therefore changes in $m^2{}_{eff}$ caused by decay or thermalization (that can be considered as changes in the background) are adiabatic and our analysis is in principle valid, irrespective of the relation among $t_d$, $t_{th}$, $t_H$. Regarding these time scales there are different cases:

[1]- $\delta t \lesssim t_d \lesssim t_{th} \lesssim t_H$. This occurs when the inequalities $\phi_0 \gtrsim 5\times 10^{-3}\frac{m}{h}$ (to have $t_d \lesssim t_{th}$), $\phi_0 \lesssim 3 \times 10^{-5}$ (to have $t_{th} \lesssim t_H$) and $(\frac{m}{h})^2 \lesssim \phi_0 \lesssim \frac{m}{h}$ (to have $t_d \lesssim t_H$) are satisfied which necessarily means $\frac{m}{h} \lesssim 10^{-2}$. In this case the following sequence of events happens: decay of the inflaton to $\chi$'s, end of the decay, and thermalization of decay products to a temperature $T \sim m$, all in a time scale shorter than the Hubble

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[5] $t_{th} = \Gamma_{th}{}^{-1}$ with $\Gamma_{th} \sim \alpha^2 \frac{n_\chi}{m^2}$ is for an out-of-equilibrium distribution of $\chi$'s. It is easily seen that for a thermal bath with $T \sim \frac{m}{2g}$ (which is the highest temperature that can be achieved) $m^{-1} \lesssim t_{th}$ also.

time. Hubble expansion then redshifts $T$ but for $\phi_0 \gtrsim (\frac{m}{h})^2$ (assuming $t_d \lesssim t_{th} \lesssim t_H$ all through the way) this sequence keeps $T \sim m$. For $\phi_0 < (\frac{m}{h})^2$ decay is no longer effective to compensate for expansion and we must wait until a later time when $\phi_0 \lesssim h^2$ and decay is completed as in the standard picture. This second stage of decay dilutes the gravitinos that are produced earlier during the first stage.

[2]- $\delta t \lesssim t_{th} \lesssim t_d \lesssim t_H$. This occurs for $\phi_0 \lesssim \ min[5 \times 10^{-3}\frac{m}{h}, 3 \times 10^{-5}]$ and $(\frac{m}{h})^2 \lesssim \phi_0 \lesssim \frac{m}{h}$ which is possible only for $\frac{m}{h} \lesssim 5 \times 10^{-3}$. This case is similar to case [1], only because of the faster thermalization the temperature is higher and closer to the maximum possible $\sim m$.

[3]- $\delta t \lesssim t_d \lesssim t_H \lesssim t_{th}$. This occurs when $\phi_0 \gtrsim \ Max[5 \times 10^{-3}\frac{m}{h}, 3 \times 10^{-5}]$ and $(\frac{m}{h})^2 \lesssim \phi_0 \lesssim \frac{m}{h}$ that is consistent only for $\frac{m}{h} \gtrsim 3 \times 10^{-5}$. In this case decay stops without effective thermalization in a Hubble time, so the distribution of $\chi$'s is out of equilibrium. Hubble expansion redshifts $m^2{}_{eff}$ as $a^{-2}$ and further decay compensates for this change. A thermal distribution of particles is not achieved, however, unless $t_{th} \lesssim t_H$ [6]. For $\phi_0 \lesssim (\frac{m}{h})^2$ the situation is as in case [1].

[4]- $t_{th} \lesssim \delta t \lesssim t_d \lesssim t_H$. This occurs when $\phi_0 \lesssim 10^{-3}\frac{m}{h}$ and $(\frac{m}{h})^2 \lesssim \phi_0 \lesssim \frac{m}{h}$ which is possible only for $\frac{m}{h} \gtrsim 10^{-3}$. In this case the decay products thermalize almost instantaneously and there is thermal equilibrium from the very beginning of the decay. Actually both $\delta t$ and $t_d$ are much greater than $\sim \frac{15}{h\phi_0}$ and $\sim \frac{m}{2h^2\phi_0{}^2}$ in this case because rapid thermalization of decay products in the thermal bath keeps the occupation number at each resonance band below one. This means that temperature can be much lower than its maximum $\sim m$ and the situation is very similar to the one in the standard picture of reheating.

Cases [1] and [2] are the most dangerous regarding the problem of gravitino production. In these cases thermal equilibrium can in principle be achieved from the very beginning and can last until the time $H^{-1} = \sqrt{\frac{3}{8\pi}} m^{-3}h^2$. Even in these cases the gravitino overproduction is not that serious since at most we have a thermal bath with temperature $T \sim m$ for a time $t \sim \sqrt{\frac{3}{8\pi}} m^{-3}h^2$, much shorter than $t \sim m^{-2}$ (assuming $m \ll (\frac{m}{h})^2$) which is the case for a radiation-dominated universe with temperature $T \sim m$. Case [3], on the other hand, is the most secure since the distribution of $\chi$'s is out of equilibrium, a distribution with less mean energy per particle and higher number density compared with a thermal distribution. Depending on the parameters of the model $m, h$ and strength of the self coupling $g^2$ one or all of these cases can happen for $(\frac{m}{h})^2 \lesssim \phi_0 \lesssim \frac{m}{h}$ but the temperature is at most of order $m$ as long as the inflaton is in this range, and is redshifted after that. Also the thermal and non-thermal corrections to the mass-squared of $\chi$ are always $\lesssim \frac{m^2}{4}$.

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[6] In this case Hubble expansion moves particles from one band to another one. This slightly lowers the time $\delta t$ spent in those bands because now there is an initial number of particles in each band. This, however, is negligible since $\delta n_\chi$ is also redshifted as $a^{-3}$.

## 3.5 Discussion

To compare our results with that of parametric amplification without the effect of final state self-interaction let us consider the case $h = 10^{-6}$,$m = 10^{-7}$, where the standard picture predicts $T_R \sim 0.1 m^{\frac{1}{2}} h = 10^8$ GeV. In the simple parametric amplification case almost all the energy density of the inflaton is converted into radiation once $\phi_0 \lesssim \frac{m}{h} = 10^{-1}$ and leads to a very high reheat temperature $T_R \sim 10^{15} - 10^{16}$ GeV which, from the point of view of gravitino overproduction, is a disaster. According to our present analysis we are in the case [3] in the above all the way from $\phi_0 = \frac{m}{h} = 10^{-1}$ to $\phi_0 = (\frac{m}{h})^2 = 10^{-2}$. This means that thermalization is not effective for $10^{-2} \lesssim \phi_0 \lesssim 10^{-1}$ and occurs much later when decay is no longer effective, so the temperature during the first stage of decay is actually lower than $\sim 10^{12}$ GeV. This results in a gravitino number density after the first epoch of decay which is too large by a factor of at most $10^{12}$. The inflaton decay is then completed via a second stage when $\phi_0 \lesssim \frac{h^2}{75} \simeq 10^{-14}$. By this time the temperature of the thermal bath and the momentum of the relativistic particles that might have been produced during the first stage are redshifted by a factor $(\frac{10^{-28}}{10^{-4}})^{\frac{1}{3}} = 10^{-8}$. The second stage will determine the effective reheat temperature to be $\sim 10^8$ GeV, and releases a large amount of entropy that dilutes the gravitinos produced during the first stage of decay by a factor of $(\frac{10^{12}}{10^8})^3 = 10^{12}$ which is now sufficient as a dilution factor.

As we noted previously, since the effect which we consider is a kinematical cutoff due to the large self-induced plasma mass of the strongly self-interacting final state decay products, we expect also to have similar effects in the broad-band case. Studies of the broad band resonance case [7, 10, 14] indicate efficient production of the decay products only for masses up to about an order of magnitude larger than the inflaton mass, so that when the broad-band resonance has built up a sufficient density of the decay products ( typically in a non-thermal "preheat" distribution) that their self-induced plasma mass exceeds this range, the decay will be suppressed. As above the decay will subsequently proceed as thermalization and Hubble expansion reduce the number densities and the induced self-mass for the decay products dips into the accessible range, resulting in a regular, quasi steady-state transfer of energy into the decay products.

Finally, we contrast the effects considered herein with those considered by Khlebnikov and Tkachev [13], who studied the semi-classical non-linear effects of the inflaton decay coupling in a massless $\lambda\phi^4$ model. Note that the effects they study have a different and independent origin from those considered here. The effects which we study are *specifically* due to final state self-interactions of the decay products which are different from, and larger (gauge strength) than, the inflaton decay coupling. If these final-state self-couplings are present, then their effects act to regulate the parametric amplification of inflaton decay.

Returning to the narrow-band case, as treated above, it is useful to ask if there are viable models where we are in the narrow-band regime from the beginning of oscillations, and for which the analysis presented here provides quantitative, as well as qualitative guidance. We note that in a simple chaotic inflation model with potential

$V(\phi) = \frac{1}{2}m^2\phi^2$, inflation ends when $V''(\phi) \simeq 24\pi V(\phi)$ which happens for $\phi \lesssim 10^{-1}$. In the case of primordial supersymmetric inflation or new inflationary models the amplitude of post-inflation oscillations around the global minimium is generically substantially smaller than the Planck scale. Depending on the parameters then, some of these models may satisfy the inequalities necessary to be in the narrow-band resonance regime, or even for the nonlinear effects to be negligible. Interestingly enough, it seems that in some viable supersymmetric models this is indeed the case, and the analysis we have undertaken here is quantitatively valid. We will return to these issues elsewhere [31].

## 3.6 Conclusion as of 1996

In conclusion, we have seen that the self-interaction of final state bosons of moderate strength, that arises very naturally in supersymmetric models, has an important impact on the decay of observable sector inflatons, besides producing a rapid thermalization rate. As a step towards improved understanding of the reheating process we have considered a simple schematic model representing generic features of supersymmetric theories, with such a final state self-coupling, and have shown that in the case of narrow-band resonance the outcome is qualitatively different from that of simple parametric amplification. Here inflaton decay occurs during two stages: one stage that consists of successive steps of partial decay, thermalization and expansion at early times which ends relatively soon, and a second stage as in the standard picture that completes the decay. In the first stage because of the quasi-adiabaticity we were able to show that the temperature and the amplitude of quantum fluctuations of final state particles are at most of the order of the mass of the inflaton (approximately), and the temperature is several orders of magnitude below the naive predictions of parametric amplification. The second stage of decay then determines the final reheat temperature and releases a substantial amount of entropy; is of particular importance in order to dilute the previously produced gravitinos in realistic supersymmetric models.

## 3.7 Conclusion as of 1999

The analytical treatment presented in this work was in the narrow-band resonance regime. In chaotic inflation models the initial amplitude of inflaton oscillations is $\phi_0 \approx 10^{-1}$ and the inflaton starts in the narrow-band regime for $10^{-6} < h < 10^{-5}$. However, for most of the inflaton coupling range $10^{-5} < h < 1$ oscillations start in the broad-band regime. At the time this paper was submitted, no work in the literature had gone through the details of broad-band resonance dynamics analytically. Therefore, a reliable quantitative estimate of the effect of final state self-interactions in the broad-band regime was not possible then. It was pointed out, however, that qualitatively the same effects were expected in that case too. There have been two important developments in this regard since then. First, lattice simulations of inflaton decay in the presence of final state self-interaction [37] showed that broad-band resonance is indeed regulated for couplings of moderate strength. Second, a detailed study of broad-band resonance [38] made similar quantitative estimates possible in this case, too. Moreover, it was shown there that for $10^{-5} < h < 10^{-3}$ the inflaton enters the narrow-band regime before the broad-band resonance becomes efficient. This implies that for $10^{-6} < h < 10^{-3}$ the narrow-band regime is the correct physical description for the termination of oscillations.

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# Chapter 4

# Parametric Resonance for Complex Fields

## 4.1 Introduction

With the advent of inflationary theories of the early universe, it has been argued that the present stage of hot FRW "big-bang" cosmology was preceded by an epoch of cosmological evolution dominated by the dynamics of scalar fields [1]. The success of inflationary models in providing explanations for flat large-scale geometry (as suggested by the location of the acoustic peaks in the CMBR anisotropies), and for the origin of approximately scale-free adiabatic density perturbations (which can be used to simultaneously fit both the CMBR anisotropies and observations of cosmic structure formation), lends support to the idea of an early scalar-field dominated epoch. A crucial question in this picture is the nature of the transition from the scalar field dominated epoch, to the hot FRW epoch, which is referred to as reheating. The nature of this transition also relates to other aspects of early universe dynamics necessary for a successful cosmology, such as mechanisms of baryogenesis, the resolution of cosmological moduli problems, and possible sources for non-thermal dark matter.

The standard approach to reheating, which applies to sufficiently weakly coupled inflaton fields [2], is to treat quanta of the inflaton field as particles, which undergo independent single particle decay; this treatment, if adequate, has the advantage that the post-inflation reheat temperature is determined by the microphysics of the model. For inflaton fields with mass as suggested by the simplest chaotic or supersymmetric models, and decaying by gravitational strength interactions, this treatment is adequate, leading to moderate reheat temperatures ( $\lesssim O(10^{10})$ GeV) which are consistent with the absence of GUT-scale defects such as monopoles, which are capable of incorporating a variety of (s)leptogenesis or electroweak mechanisms for generation of the BAU, and which avoid, in the supersymmetric case, cosmological problems with thermal overproduction of gravitinos after reheating [3].

Recently it has been realized that the standard treatment of reheat in terms of single particle inflaton decay may be seriously misleading in circumstances where there is coherent enhancement of the transition to bosonic decay products [4]-[16]. For large mode occupation numbers of the decay product field we may treat its dynamics as being essentially classical. Mode by mode for the decay product field its coupling to the oscillating inflaton field induces a periodic time dependence in the mode mass (modulated by cosmic expansion which "sweeps" the time dependence of each comoving mode through the bands of the stability chart of the mode equations.) This periodic modulation of the parameters of the oscillator associated with each mode of the decay product field, can induce parametric resonance in bands of the mode parameters, leading to exponential growth in the decay mode amplitude. The resonant decay of the inflaton may have important cosmological implications like non-thermal symmetry restoration and subsequent formation of topological defects [17, 18], revival of GUT baryogenesis scenarios [19, 20], supersymmetry breaking [21], superheavy particle production [16, 22], and gravitino production [23, 24].

The exponential growth in the mode occupation number may be modified or regulated by a number of physical processes. These include the decay of produced quanta to other particles [20, 25] or the rescattering of final state particles [9]. Another possibility, occurring in models with gauge-strength self-interactions between the produced final state particles, is the regulation of the parametric resonance by the self-interaction induced effective masses of the produced quanta, which can move these quanta out of the available resonance bands; in this case, resonance only proceeds as thermalization and Hubble dilution reduce the plasma masses of the final state quanta, resulting in a quasi-steady-state resonance conversion of inflaton oscillation energy to decay products [26]. This general scenario for the physically realistic case of decay products with gauge charge has been verified in explicit calculations in the narrow-band resonance case [26], and in numerical simulations in the broad-band case [10].

While to date analytical and numerical treatments of parametric resonance have considered oscillations of a single real field decaying to a single real decay product field, in realistic models the field content is often more extensive. In the case of supersymmetric theories this is unavoidably the case, as the physical scalars of simple (N=1) supersymmetry come as components of chiral multiplets and are complex. So for these types of theories, we should at the very least consider the nature of coherent decays when the fields involved are complex, though non-supersymmetric models with multiple real scalar fields may share some of the features of the simplest complex case.

Within supersymmetric models of particle physics, there are several different circumstances under which the decay of a homogeneous complex scalar condensate may occur in the early universe. At the end of inflation one expects to have a spatially homogeneous inflaton scalar condensate, whose decay energy will ultimately be responsible for cosmic reheating and the initiation of hot big-bang cosmology. As well, in the supersymmetric standard model there are directions in the scalar field space of squarks and sleptons which are F-flat and D-flat, and which only gain a potential from supersymmetry breaking. In the early universe these directions may be populated with (very) large vev's after the end of the inflationary epoch; these vev's may carry enormous vev to mass ratios (Mathieu resonance parameter $q$) and couple to other directions in scalar field space with couplings capable of inducing resonant decay. Finally, supersymmetric models are generically plagued with gauge singlet scalar moduli, whose homogeneous oscillation poses grave cosmological difficulties which might be ameliorated by coherent decay of their oscillation amplitude.

For self-interactions of complex scalars of the general form dictated by the F-term and D-term couplings arising in globally supersymmetric theories, the fields generally appear in complex conjugate pairs for the F-terms and diagonal D-terms. For example, let us consider a complex scalar field $\Phi$ in a chiral supermultiplet whose decay will be induced by a trilinear (renormalizable) coupling in a superpotential $W$ to a chiral multiplet labelled by its scalar $\Xi$, where $W \supset \frac{1}{2}g\Phi\Xi\Xi$. The resulting F-term coupling inducing the decay is then of the form $g^2\Phi^*\Phi\Xi^*\Xi$, and is invariant under global phase redefinitions of either the $\Phi$ or the $\Xi$. We will see in the next section that in cases such as this the phase invariance of the resulting couplings implies that the equations for modes of the real and imaginary components of $\Xi$ are decoupled and independent, and will allow us to simply analyze the resonant decay of a $\Phi$ condensate with out of phase oscillations for the real and imaginary components of $\Phi$, into real and imaginary components of the decay product field $\Xi$.

We can always phase rotate our scalar field $\Phi$ to a basis such that its initial vev lies along the real axis. If there is no component of force along the direction of the imaginary axis (i.e. the scalar potential is phase invariant), the trajectory of the motion of $\Phi$ is limited to the real axis and the field hits the origin as it oscillates back and forth. In this case, provided that the coupling of the oscillating field to the final state field is also phase invariant, the situation is exactly that of a real oscillating field, and the same arguments apply for parametric resonance particle production. However, if the scalar potential is not phase invariant, i.e. depends on the phase of the oscillating field as well, a torque is exerted on the field. This leads to the deflection of the trajectory from a straight line and results in changing the trajectory into something that finally resembles an ellipse, after the torque in field space has effectively ceased its action. In this case the field no longer passes through the origin but rather has a finite distance of closest approach to it. This will have important implications for broad-band parametric resonance as we discuss below.

In supersymmetric models not only are complex scalar fields inherently involved, but also a phase dependent part of the scalar potential can arise naturally from supersymmetry breaking. Let us consider the simplest case with the following terms only involving the inflaton in the superpotential $W = \frac{1}{2}m\Phi^2 + \frac{1}{3}\lambda\Phi^3$. In supergravity models with broken supersymmetry, there is a corresponding phase dependent term (the "A-term") $Am_\Delta \frac{\partial W}{\partial \Phi}$ + h.c. in the scalar potential, where $A$ is a dimensionless model-dependent constant and $m_\Delta$ is the scale of supersymmetry breaking in the sector in which $\Phi$ lies. There is also a phase dependent term $m\lambda^*\Phi\Phi^{*2}$ + h.c. in the F-term part of the scalar potential. This generically occurs in minimal supergravity models for inflation where the superpotential contains a series of $\lambda_n \frac{\Phi^n}{M^{n-3}}$ terms [27], and occurs even in "no scale" supergravity models after the inclusion of radiative coreections to the effective potential [28].

For $V \supset \Phi^m(\Phi^*)^n$ the potential along the angular direction is periodic with $m - n$ minima. In general, during inflation $\Phi$ rolls down to its minimum both along the radial and angular directions. In order to have a torque to deflect the trajectory, $\Phi$ must not be at the minimum along the angular direction at the onset of radial motion. This can happen in two ways: either there are several phase dependent parts of the potential with a non-adiabatic transition from the minimum of one to another, or $\Phi$ does not settle at the minimum along the angular direction. The first possibility happens when other supersymmetry breaking sources in the early universe (e.g. non-zero energy density of the universe or finite temperature effects) are dominant over the low energy one. In this case the minimum in the angular direction at early times is different from that at late times (due to independent phases for the coefficients of different A-terms). If the transition from one minimum to another one is non-adiabatic, $\Phi$ will not be at the minimum at late times regardless of its start at the minimum at early times. The second possibility happens when the potential along the angular direction is flat enough during inflation. In this case $\Phi$ will not roll down to its minimum and can start at any position at the onset of radial motion. In both cases, the further $\Phi$ is away from the minimum, the larger the deflection of its trajectory and the wider the ellipsoidal shape will be.

## 4.2 Complex Mathieu Resonance

As described in the previous section, the potential for complex scalar oscillations usually includes (as well as the scalar mass-squared terms) Hubble induced A-terms which are mainly effective during the first few cycles of oscillation. The A-terms provide a "torque" to the complex oscillation during the first few cycles, resulting in a net "elliptic" motion in the mass-term induced potential, after the A-terms have effectively ceased to be active. The resulting elliptic oscillation in the mass term potential will be damped by the Hubble drag, resulting in the ellipse shrinking over time.

In order to introduce new considerations characteristic of resonance with complex fields, without getting involved in model dependent details, in most of this chapter we shall simply ignore the damping and consider complex, elliptic, constant amplitude oscillations. In particular, this means we do not need to specify which particular field is considered (e.g. inflaton versus susy standard model flat direction), nor do we need to determine the cosmological details involved in determining expansion and damping at the time that the oscillations of the field in question occur. In addition to presenting a tractable and interesting mathematical problem, consideration of the undamped oscillation should also provide the essential features of the full cosmological case including the effects of expansion, at least in the generic case of broad-band resonance. This follows from the key observation of [4, 5] that in the broad-band case the resonant excitation of the decay product field occurs over a tiny fraction of the cycle of the driving field, when the latter passes near the origin, as only here do the decay product field dynamics depart from the adiabatic regime. So mode number excitation proceeds by a series of abrupt jumps, and the dynamics of a given jump may be considered with the instantaneous value of the oscillator parameters, resulting in the picture of "stochastic resonance" analyzed in [5]. The present paper discusses the changes in the dynamics of decay mode excitation which arise from the complex nature of the driving field oscillation—the differences in question arise from suppression of the adiabaticity violations that induce the jump in mode occupation numbers of the decay product field, so we expect that considerations using the instantaneous value of the driving oscillation amplitude should give insight in the complex case, much as such considerations did in the real stochastic resonance case.

In any case, for the purposes of our present calculations we shall consider the parametric resonance production of decay product field modes $\Xi$, from phase-invariant coupling to constant-amplitude out of phase ("elliptic") oscillations of a driving field $\Phi$. Detailed cosmological studies of applications to inflaton or moduli oscillations will be considered elsewhere.

With the couplings discussed in the previous section, after the A-terms cease to be effective, the equation of motion for the $\Xi$ field is of the form:

$$\ddot{\Xi}_k + \left(\frac{k^2}{a^2} + m_\Xi^2 + g^2|\Phi|^2\right)\Xi_k = 0, \tag{4.1}$$

where $\Xi_k$ is the decay product field mode with comoving wavenumber $k$, $a$ is the FRW scale factor, $m_\Xi$ the mechanical mass of the $\Xi$, and the superpotential coupling is as above. We note that both the real and imaginary piece of the $\Xi$ field will separately obey this equation, and hereafter we use $\chi$ to denote either the real or imaginary piece of $\Xi$.

In our analysis, we will treat the physical momentum of the decay field mode and the relative phase and amplitude of the driving field oscillation as fixed parameters, and attempt to map out the regions of instability in their parameter ranges. As noted above, for the case of stochastic broad-band resonance, where the amplification occurs in small intervals while the field passes close to the origin, one should be able to approximate the instantaneous behaviour within each interval by the corresponding behaviour of a system of the type we analyze here.

We decompose the driving field $\Phi$ into real and imaginary pieces as follows:

$$\Phi = \phi_R + i\phi_I. \tag{4.2}$$

By a phase rotation we put the largest amplitude component of oscillation into the real piece, and so we write:

$$\phi_R = \phi \sin(m_\phi t) \tag{4.3}$$

$$\phi_I = f\phi \cos(m_\phi t), \tag{4.4}$$

where now $\phi$ is the constant amplitude of the real component of oscillation and $f \in [0,1]$ is the "out of phase" fractional component giving elliptic oscillation in the complex $\Phi$ plane; we will be particularly interested in the case where $f \ll 1$.

We wish to cast this into the canonical form of the (real) Mathieu equation:

$$y'' + (A - 2q\cos(2z))y = 0, \tag{4.5}$$

where $'$ denotes derivative with respect to the independent variable $z$. We begin by substituting our definition of the $\Phi$ field into (4.1) above, giving

$$\ddot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2 + g^2\phi^2\left(\sin^2(m_\phi t) + f^2\cos^2(m_\phi t)\right)\right)\chi_k = 0. \tag{4.6}$$

We replace $\cos^2(m_\phi t)$ with $1 - \sin^2(m_\phi t)$ and collect terms, giving

$$\ddot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2 + f^2g^2\phi^2 + \left(1 - f^2\right)g^2\phi^2\sin^2(m_\phi t)\right)\chi_k = 0. \tag{4.7}$$

Using the half-angle formula $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$ we obtain the form

$$\ddot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2 + f^2g^2\phi^2 + \frac{1}{2}\left(1 - f^2\right)g^2\phi^2\left(1 - \cos(2m_\phi t)\right)\right)\chi_k = 0. \tag{4.8}$$

which may be rewritten in the form of the Mathieu equation

$$\chi_k'' + (A_k(f) - 2q(f)\cos 2z)\,\chi_k = 0 \tag{4.9}$$

with the following new identifications:

$$z = m_\phi t \tag{4.10}$$

$$A_k(f) = \frac{\frac{k^2}{a^2} + m_\chi^2 + f^2 g^2 \phi^2}{m_\phi^2} + 2q(f) \tag{4.11}$$

$$q(f) = \frac{(1 - f^2) g^2 \phi^2}{4 m_\phi^2}. \tag{4.12}$$

Notice that the coefficients of the Mathieu equation are now functions of the imaginary fraction $f$. This is an important feature, as it means that the characteristic behaviour of the parametric resonance as described by the Mathieu equation takes the same form in the complex case as in the real case; however, *the relationship between the physical parameters of the process and the Mathieu coefficients is redefined.*

So we see that there is a mapping that takes the Mathieu equation for the complex modes of the decay field when driven by the complex parametric field with out of phase real and imaginary pieces in its oscillation amplitude, and maps it to a real Mathieu equation for the oscillations of the real and imaginary pieces of the decay product field with shifted parameters. There are several features of this mapping that simply encapsulate physical features of the original problem.

First we note that for the case where $f = 0$, where the oscillation of the parametric driving field is along the real axis, the coefficients $A_k$ and $q$ reduce to those previously considered in the literature for the case of purely real parametric oscillation [4, 5]. In the other extreme limit, when $f = 1$, we have $q(1) = 0$, meaning that one is restricted to be along the $A_k$ axis on the Mathieu equation stability diagram, allowing only non-resonant particle production. This corresponds to the physical observation that because the decay couplings were phase invariant, our original equation for the complex oscillations involved only the magnitude of $\Phi$ in the oscillation equation. In the case that the real and imaginary pieces of the $\Phi$ oscillation have the same amplitude ($f = 1$), then the coefficients in the $\chi$ equation of motion become time independent and there can be no parametric amplification. We also note that the imaginary fraction $f$ of the oscillations enters into the coefficients only as $f^2$, meaning that the effect is the same regardless of direction traveled around the ellipse (i.e. whether $f$ is positive or negative). This reflects the fact that the original equation for the complex oscillations is second order and symmetric under time inversion, which corresponds to having the parametric field circulate about the oval in the $\Phi$ plane in the opposite sense, or reversing the sign of $f$.

Finally, in the intermediate regime $f \in (0,1)$, $q(f)$ is always decreased, while $A_k$ is always increased (compared to $f = 0$), so increasing $f$ never causes the system to leave the physical regime. As noted above, increasing $f$ means moving "inland" on the stability diagram for the Mathieu equation. In general, this causes suppression of the resonant growth of the $\chi$ modes; however, it also allows one to explain the counterintuitive observation that in certain cases the resonant band exponential growth parameter $\mu_k$ may actually increase as one turns on the out of phase component $f$. To understand this, imagine that for oscillations with no imaginary fraction $f$ one is sitting in parameter space at the lower border of one of the instability bands (where $\mu_k$ is zero). Now slowly increase $f$. The parameter mapping derived above implies that you start to move in a "northwest" direction on the band chart into the instability band. As such, your $\mu_k$ begins to increase. As you continue to increase $f$, you will eventually hit the maximum possible $\mu_k$ along your trajectory, after which your $\mu_k$ begins to drop again. Eventually you will leave the instability band altogether for sufficiently large $f$. For very high order instability bands, it should be possible to encounter many bands along one trajectory of increasing $f$.

From a different point of view, were one to look at the instability diagram as a function of the $A_k$ and $q$ of standard real Mathieu parametric resonance (ie. as a function of our $A_k(0)$ and $q(0)$, for different values of $f$), the effect of turning on $f$ would be seen to manifest itself as both a narrowing and a downwards shift of the instability bands as $f$ increased. In addition, isocontours of $\mu_k$ would be seen to "flow out" of the bands as $f$ increases. For $f = 1$ each instability band collapses to a line with $\mu_k = 0$, as with a phase independent coupling of the parameter field to the mode field there would be no time dependence in the mode field equation of motion, and the system would be stable for all $A_k$ and $q$. Figure 4.1 illustrates in detail both the bending of the bands and the decrease of $\mu_k$ for the first resonance band as we turn on the imaginary fraction $f$ of our oscillations. We expect that the suppression of resonance for fixed non-zero imaginary fraction $f$ is stronger in higher resonance bands, as at larger $q$ the resonance proceeds by violation of adiabaticity in the decay mode evolution, and for large $q$ at fixed $f$ the induced decay mode field mass is always large. This is illustrated in Figure 4.2 where we show the quenching of resonance for the first 7 resonance bands, as $f$ is turned on. We see that there is more severe suppression for the higher bands, in accord with our physical intuition; in the next section we will analytically estimate the extent of the domain of surviving resonant bands, for fixed imaginary fraction $f$.

## 4.3 Parameter Domain For the Broad-Band Resonance

Let us first briefly discuss the effect of mixing in the narrow-band regime. The narrow-band resonance in the case of a real oscillating field is efficient for $(\frac{m}{g})^{\frac{4}{3}} \leq \phi \leq \frac{m}{g}$ [6]. In the case of a complex field with mixing parameter $f$ this reads as

$$\left(\frac{m^2 + f^2 g^2 \phi^2}{(1-f^2)g^2}\right)^{\frac{2}{3}} \leq \phi \leq \left(\frac{m^2 + f^2 g^2 \phi^2}{(1-f^2)g^2}\right)^{\frac{1}{2}}, \qquad (4.13)$$

where $m$ and $g$ are replaced with $\sqrt{m^2 + f^2 g^2 \phi^2}$ and $\sqrt{1-f^2}g$, respectively. It is easily seen that the condition for an efficient narrow-band resonance remains almost unchanged in the complex case, unless $f$ is close to 1. Therefore, in physically interesting situations the narrow-band resonance will not be substantially affected by the mixing. Of course, in the extreme case with $f = 1$, there is no time variation in the Mathieu equation and, hence, no narrow-band resonance.

In the case of real broad-band parametric resonance, Kofman, Linde, and Starobinsky [4, 5] argue that the requirement of adiabaticity violation for broad-band parametric amplification implies that it only occurs with significant $\mu_k$ for $A_k - 2q \lesssim \sqrt{q}$. Their argument presupposes that the decay terminates after it has entered an "explosive" phase, where the effective mass of the decaying $\phi$ is dominated by the coupling to the plasma of decay product $\chi$ modes which has been built up by parametric resonance decay. The effective physical 3-momentum of the quasi-relativistic decay $\chi$ modes is of order their energy, which is no more than of order the induced mass of the decaying $\phi$; this is of order $g\chi_{\rm end}$ which in turn is of order $g\phi_{\rm end}$ which can also be written $\sqrt{g m_\phi^{\rm eff}\phi_{\rm end}}$. So $(k^2_{\rm phys}/m_\phi^2) \lesssim (g\phi_{\rm end}/m_\phi^{\rm eff})$, which can be rewritten as $A_k - 2q \lesssim \sqrt{q}$. For a detailed discussion we refer to the treatment in [5].

We have seen in the preceding section that the case of complex oscillation with imaginary fraction $f$ can be mapped onto a Mathieu equation with shifted resonance parameters. By substituting the "shifted" parameters induced by the non-zero imaginary component of oscillation, we should thus be able to determine what range of $q$ supports broad-band resonance for oscillation with a given fraction of out of phase imaginary component for the oscillation of the driving parameter.

Recall the expressions for the equivalent shifted $A_k(f)$ and $q(f)$ from equations (4.11) and (4.12) respectively. Substituting these expressions into the relation $A_k - 2q \lesssim \sqrt{q}$ allows us to write it as:

$$\frac{\frac{k^2}{a^2} + m_\chi^2 + f^2 g^2 \phi^2}{m_\phi^2} \lesssim \frac{\sqrt{(1-f^2)}\, g\phi}{2m_\phi}. \tag{4.14}$$

This leads us to the relation

$$A_k(0) - 2q(0) + 4f^2 q(0) \lesssim \sqrt{1-f^2}\sqrt{q(0)}, \tag{4.15}$$

or, defining $E_k \equiv A_k(0) - 2q(0)$, we have

$$E_k + 4f^2 q(0) \lesssim \sqrt{1-f^2}\sqrt{q(0)}. \tag{4.16}$$

If we recall that physical values of $E_k$ are positive semi-definite, we find that for a fixed non-zero imaginary fraction $f$ there is an upper bound on the parameter $q(0)$ for which resonance occurs, and the allowed range of resonant $q(0)$ is bounded above as $\frac{1-f^2}{16f^4}$. (For the small imaginary fractions $f$ of physical interest, however, the weaker approximate bound of $\frac{1}{16}f^{-4}$ will suffice). So instead of an an ever widening resonance region above the $A_k = 2q$ line, with thickness of order $\sqrt{q}$, as one has in the real case, in the complex case with fixed non-zero imaginary fraction $f$, one instead has a region above the $A_k = 2q$ line of finite extent, with an upper bound on the values of the $q$ parameter which can result in resonance. This is qualitatively reasonable, as a fixed imaginary fraction $f$ for the oscillation means that as we scale up $q$ the ellipse of $\Phi$ broadens as it lengthens, preserving its shape; so throughout the $\Phi$ oscillation $|\Phi|$ has a large value, inducing a large mass for the modes of the decay field $\Xi$, which in turn leads to adiabatic evolution of the $\Xi$, and suppression of broad-band parametric resonance production of the $\Xi$.

## 4.4 Complex Resonance in "Instant Preheat"

Recently, a simpler method of efficient scalar field decay has been proposed, called "instant preheat" [16]. Within models of this type the decaying field rolls once through the origin, at which point the mass of the decay product field to which it is coupled passes through zero, and modes of the decay product field experience non-adiabatic excitation. As the decaying field rolls away from zero (perhaps monotonically) the modes of the decay product field grow in mass; they drain energy from the decaying field through their mass. As the mass of the modes of the decay product field grows, so does their decay width; their subsequent decay, after their mass and decay width have grown sufficiently, then releases the energy they have taken from the original decay field, and dumps it into their final decay products, which thermalize the resulting energy.

It is interesting to note that while final state effects such as rescattering, backreaction, or plasma masses can prevent preheating from occurring, they are unimportant in the instant preheating scenario. The reason is that for these effects to become important, (at least) several oscillations are needed to build up a large enough occupation number for the final state field. In the instant preheating, on the other hand, the energy drain from the oscillating field occurs during each half of an oscillation. In fact, instant preheating can be efficient even if the adiabaticity condition is violated only during the first half of the first oscillation. Therefore, instant preheating is essentially unaffected by the final state effects. The mixing of the real and imaginary parts of the oscillating field, on the other hand, has the same effect in the instant preheating case as in the standard preheating scenario. We recall that the torque from A-terms deflects the trajectory of the oscillating field from that of a straight line. The Hubble induced A-terms have their largest value at the beginning of the oscillations, and rapidly decrease with Hubble expansion. This means that the deflection is largest during the initial oscillations. Thus, a large enough $f$ to restore adiabaticity in the preheating case could do the same for the instant preheating case.

## 4.5 Non-Convex Potentials

Another possibility to achieve rapid decay of a homogeneous condensate occurs in the case that the potential governing the evolution of the condensate scalar has non-convex behaviour over some region of field space [12, 29]. In this circumstance, it becomes energetically favorable for a scalar condensate in the non-convex region to decompose into inhomogeneous modes; provided the inhomogeneity occurs over long enough wavelengths, the price one pays in kinetic energy for the inhomogeneity is more than compensated by the decreased average potential energy of the regions of field excess and deficit compared to the average field value. This produces a wavenumber band for exponential growth of the mode amplitudes. This has been considered in both the case of inflaton decay [12], and in the case of the growth of inhomogeneities in scalar condensates corresponding to F-flat and D-flat directions of the standard model with non-convex potentials of the type arising from gauge-mediated supersymmetry breaking [29]. It is clear that this is one type of instability which is not vitiated by having the scalar order parameter complex or involving multiple scalar fields. If there is a region in field space with respect to which the potential is non-convex in some direction, then fluctuations corresponding to modes of the field variation in that field direction, of sufficiently long wavelength, will win on the potential versus kinetic energy budget, and grow exponentially. Indeed the treatment of (complex) flat directions in the supersymmetric standard model in [29] explicitly analyzes the conditions for instability of a complex field with a potential which is a non-convex function of the field modulus, and exhibits the resulting instability bands.

## 4.6 Other Couplings

Here, we briefly comment on the situation for another type of coupling between $\Phi$ and $\Xi$ fields which is also of interest and application. This is the $g^2(\phi_R\chi_R + \phi_I\chi_I)^2$ coupling whose simplest manifestation is for the potential $V(\Phi) = \frac{1}{4}\lambda|\Phi|^4$, with $\Phi$ and $\Xi$ being the same field. It also arises in supersymmetric models from the D-term part of the scalar potential. This type of coupling leads to the mixing of $\chi_R$ and $\chi_I$ mode equations:

$$\ddot{\chi}_{R,k} + \left(\frac{k^2}{a^2} + m_\chi{}^2 g^2 \phi_R{}^2\right)\chi_{R,k} + g^2\phi_R\phi_I\chi_{I,k} = 0 \tag{4.17}$$

$$\ddot{\chi}_{I,k} + \left(\frac{k^2}{a^2} + m_\chi{}^2 g^2 \phi_I{}^2\right)\chi_{I,k} + g^2\phi_R\phi_I\chi_{R,k} = 0. \tag{4.18}$$

In this case the mass eigenstates are $\frac{\phi_R\chi_R+\phi_I\chi_I}{\phi}$ and $\frac{\phi_I\chi_R-\phi_R\chi_I}{\phi}$ instead of $\phi_R$ and $\phi_I$ themselves. For oscillatory motion of $\phi_R$ and $\phi_I$ with a phase difference, there are two periodic changes that may lead to resonance: change in the mass eigenvalues (the usual parametric resonance) and change in the mass eigenstates. They can't be simply superimposed and it is not very easy to give rough arguments for the instability bands and the respective value of $\mu_k$'s. The important point is that for such a coupling, even in the $f = 1$ case there is still time variation in mode equations. This variation is present in both the mass eigenstates and mass eigenvalues.

## 4.7 Cosmic Expansion and Complex Resonance

So far, we have considered modifications to parametric resonance decay which arise in complex field oscillations in the absence of effects of Hubble expansion. As we noted above, since broad-band resonance is induced by non-adiabaticity of the $\chi$ evolution during small intervals of the $\phi$ oscillation, instantaneous approximation of the $\chi$ excitation should be a useful guide during each of the jumps in mode number. Cosmic expansion then functions to shift the parameters of the oscillator between episodes of mode excitation as $\phi$ passes near zero. We now examine the implications of this in both the narrow- and broad-band cases.

Implications for the narrow-band case are simple; as we have seen, the introduction of a phase difference between real and imaginary components of our complex inflaton field $\Phi$ only kills narrow-band resonance for phase differences approaching $\frac{\pi}{2}$, or, in the language of this paper, for $f \cong 1$. Therefore, the resonance should be qualitatively the same in the static approximation and with the Hubble expansion included.

For broad-band resonance the situation is completely different. According to equation (4.16), the broad-band resonance is shut off for $q > \frac{1}{16}f^{-4}$. In the static limit $f$ and $q$ are both constant and resonance is either suppressed, or viable. In an expanding universe, $f$ eventually becomes approximately constant as the Hubble induced A-terms turn off (indeed, as pointed out earlier, after several Hubble times the motions along the real and imaginary directions are decoupled and free), while, on the other hand, $q(t) = (\frac{g\phi(t)}{2m})^2$ is redshifted as $a^{-3}$. This implies that even if the resonance is suppressed initially, it may be initiated after a sufficient time such that $q(t) < \frac{1}{16}f^{-4}$. The right-hand side is less than 1 (or very close to it) for $f \gtrsim \frac{1}{2}$. Therefore, in the case of large out of phase components of oscillation, the broad-band resonance is killed and resonance may resume only in the narrow-band regime at a later time. In most physically interesting cases, however, $f < \frac{1}{2}$ and the right-hand side is considerably greater than 1. In such cases, broad-band resonance is not eliminated in an expanding universe, but rather delayed. The parameter $f$ is determined by the action of the scalar potential (including A-terms) for the oscillating field, and after the initial oscillations it often becomes effectively time-independent. Depending on the dimensionality of the A-term, it may be a function of $q_i$, the value of $q$ at the start of oscillations. If $q_i < \frac{1}{16} f^{-4}$, the onset of broad-band resonance will be unaffected. For $q_i > \frac{1}{16} f^{-4}$, resonance is delayed initially, but will resume after sufficient expansion such that $q < q_{\rm eq} = \frac{1}{16} f^{-4}$. A larger $f$ leads to a smaller $q_{\rm eq}$, for $f \gtrsim \frac{1}{2}$ we have $q_{\rm eq} < 1$ and resonance can only occur in the narrow-band regime. For $f = 1$ resonance is eliminated.

Even though the broad-band resonance (for interesting cases) is only delayed in an expanding universe, the mixing still has important consequences. Perhaps the most notable example relates to the production of superheavy particles during resonance. In the standard preheating, $\Xi$'s with a mass up to $q^{\frac{1}{4}} m_\phi$ can be produced. A reduction in $q$ at the onset of resonance implies a reduction in the maximum mass of produced particles. This is even more pronounced in the instant preheating case. Here $\Xi$ decay products $\Psi$ with masses up to $q^{\frac{1}{2}} m_\phi$ and which have a large enough coupling $h$ to $\Xi$, can be produced [16]. A smaller $q_{\rm eq}$ has a two-fold effect in this case. First, the allowable masses are smaller, and second, the decay rate $\Gamma_d = \frac{h^2}{8\pi} g\phi$ may not be large enough (compared to the frequency of oscillations $m_\phi$) for efficient production of $\Psi$'s. It is easily seen that $\Gamma_d \leq m_\phi$ for $h \lesssim 4\pi^{\frac{1}{2}} f$. Therefore, $\Psi$ production is not efficient if $h \lesssim 4\pi^{\frac{1}{2}} f$. Even for $h \gg 4\pi^{\frac{1}{2}} f$, only $\Psi$'s with a mass $m_\psi \lesssim \frac{1}{4f^2} m_\phi$ can be produced.

## 4.8 Conclusions

In this chapter, we have considered the changes to the standard picture of parametric resonance decay of a real homogeneous cosmological scalar field which arise if the field is instead complex, with out of phase oscillation of its real and imaginary components and a phase invariant decay coupling. For the case of complex Mathieu type resonance, we give an explicit mapping to a corresponding real Mathieu resonance with shifted parameters that encode the effects of the out of phase components of the oscillating decay field. We showed the resulting effects on the instability bands, demonstrating how they shift and shrink with increasing out of phase ("elliptic") component of the driving field motion, limiting the swath of instability to a finite area on the $A_k$-$q$ chart, and eliminating broad-band resonance in the higher modes. We argued that similar effects may be present in the case of complex field models of "instant preheat," but that instabilities due to regions in field space with non-convex potentials are qualitatively the same in the complex case. Finally, in the context of an expanding FRW universe, we noted that the presence of a fraction of out of phase oscillation would usually delay the onset of parametric resonance, but not eliminate it entirely.

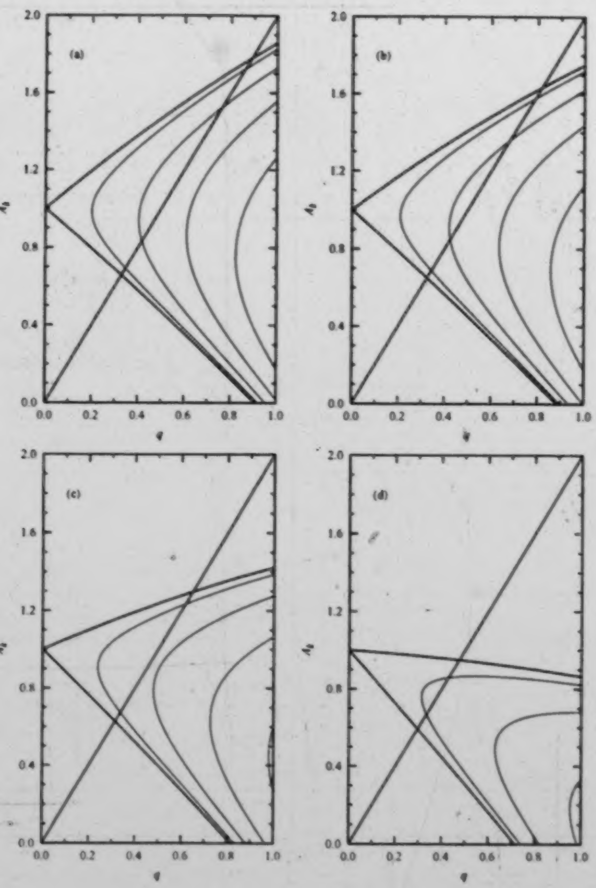

Figure 4.1: The Mathieu equation stability diagram for the first resonance band. The physical region lies above and to the left of the diagonal line $A_k = 2q$. Contour lines represent isocontours of $\mu$ starting from $\mu = 0$ (band boundary) and increasing by units of 0.1 as one moves from left to right in the band. Different panels represent different imaginary fractions $f$: (a) $f = 0$; (b) $f = 0.2$; (c) $f = 0.4$; (d) $f = 0.6$.

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# Chapter 5

# Reheating and Supersymmetric Flat-Direction Baryogenesis

## 5.1 Introduction

Initially, one of the most attractive features of Grand Unified Theories (GUTs) [1] was the prospect that they might provide an explanation [2] for the matter-antimatter asymmetry of the Universe, via their new interactions that violate baryon and/or lepton number. Subsequently, it has been realized that, even in the Standard Model, at the non-perturbative level there are sphaleron interactions that violate both baryon and lepton number. This discovery has given rise to new scenarios for baryogenesis, at the electroweak phase transition [3] or via leptogenesis followed by sphaleron reprocessing [4]. Supersymmetric extensions of the Standard Model offer yet more scenarios for baryogenesis. For example, they may facilitate electroweak baryogenesis by permitting a first-order electroweak phase transition despite the constraints imposed by LEP [5]. There is also the possbility that they may contain perturbative interactions that violate baryon and/or lepton number via a breakdown of $R$ parity, which under certain circumstances [6] can induce baryogenesis.

However, perhaps the most attractive mechanism offered by supersymmetry is that proposed by Affleck and Dine, according to which [7] a condensate of a combination of squark and/or slepton fields may have formed during an inflationary epoch [8] in the early universe, causing the vacuum to carry a large net baryon and/or lepton number, which is then transferred to matter particles when the condensate eventually decays. We recall that the condensate forms along some flat direction of the effective potential of the theory, which we take to be the Minimal Supersymmetric extension of the Standard Model (MSSM) at low energies. In the conventional approach to Affleck-Dine baryogenesis, the condensate is essentially static until a relatively late cosmological epoch, when it starts to oscillate. In turn, the termination of the period of oscillation has been calculated in terms of the magnitudes of the soft supersymmetry-breaking terms present in the effective potential, which become significant only at low temperatures, and of the thermalization effects of inflaton decay [9].

The purpose of this chapter is to re-examine this Affleck-Dine mechanism by incorporating a more complete treatment of the reheating of the universe after the inflationary epoch. We argue that the flat directions are in general coupled to other fields that are kinematically accessible to inflaton decay. These fields therefore have non-trivial statistical densities, and become thermalized. The couplings of these densities to the flat directions induce effective supersymmetry-breaking masses and $A$ terms for the erstwhile flat fields. As a result, the 'flat' directions start oscillating earlier than previously estimated. Subsequently, the oscillations also terminate earlier, as the flat-direction condensate interacts with the plasma of inflaton decay products and evaporates. The bottom line is that previous estimates of the resulting baryon/lepton asymmetry of the universe may be substantially altered, and we estimate some orders of magnitude for different representative parameter choices.

## 5.2 Flat Directions

The $D$-flat directions of the MSSM are classified by gauge-invariant monomials in the fields of the theory. These monomials have been classified in [10], and, for directions which are also $F$-flat for renormalizable standard model superpotential interactions, the dimension of the non-renormalizable term in the superpotential which first lifts the respective $D$-flat direction has also been derived. Hereafter, we consider only those $D$-flat directions which are not lifted by renormalizable superpotential interactions. These correspond to 14 independent monomials, and each monomial represents a complex $D$-flat direction: one vev magnitude and one phase (all fields in the monomial have the same vev). Since the monomials are gauge-invariant, appropriate gauge transformations generated by non-diagonal generators can be used to remove that part of the $D$-term contribution to the potential which comes from the non-diagonal generators. Also, any relative phase among the fields in the monomial can be rotated away by those gauge transformations which are generated by diagonal generators. There remains only one overall phase, i.e., the phase of the flat direction, which can be absorbed by redefinition of the scalar fields. Note that the $D$-term and $F$-term parts of the scalar potential are invariant under such a redefinition (which is equivalent to a U(1) symmetry transformation) while the soft-breaking terms and fermionic Yukawa terms generally are not. So we can always arrange the vev of the fields in the monomial to be initially along the real axis. We also note that such a non-zero vev breaks spontaneously the MSSM gauge group.

As an explicit example, consider the simplest case, which is the $H^u L$ flat direction. If the $T_3 = \frac{1}{2}$ component of $H^u$ and the $T_3 = -\frac{1}{2}$ component of $L$ have the same vev, then all the $D$ terms from both diagonal and non-diagonal generators of the MSSM are zero. The non-diagonal ones are identically zero and the equality of the vev makes the diagonal ones zero as well. These vev's can then be chosen along the real axis as noted above. There are eight real degrees of freedom in the $H^u$ and $L$ doublets. Two of them comprise the flat direction and another three are Goldstone bosons eaten by the gauge fields of the spontaneously-broken symmetries. The remaining three are physical scalars which are coupled to the flat direction, and are massive due to its vev.

Now that since all fields in the monomial have the same vev and are real, by an orthogonal transformation we can go to a new basis where there is only one direction with a non-zero vev. Let us label this direction $\alpha$ and the orthogonal directions generically as $\phi$. Therefore $\alpha_R \neq 0$ while $\alpha_I = \phi_R = \phi_I = 0$. For the specific $H^u L$ example, these are the following combinations after the Goldstone bosons are absorbed by the Higgs mechanism:

$$\begin{aligned}
\sqrt{2}\alpha_R &= (H_1)_R + (L_2)_R \\
\sqrt{2}\alpha_I &= (H_1)_I + (L_2)_I \\
\sqrt{2}\phi_1 &= (H_1)_R - (L_2)_R \\
\sqrt{2}\phi_2 &= (H_2)_R - (L_1)_I \\
\sqrt{2}\phi_3 &= (H_2)_I + (L_1)_R
\end{aligned}$$

The $D$ terms from the $T_3$ and $U(1)_Y$ generators give terms $g^2\alpha^2{\phi_1}^2$ (up to numerical factors) in the potential, whilst those from $T_1$ and $T_2$ give $g^2\alpha^2{\phi_2}^2$ and $g^2\alpha^2{\phi_3}^2$ terms (up to numerical factors). It is a generic feature that all fields entering in the flat direction monomial which are left after the Higgs mechanism (except the linear combination which receives the vev after diagonalization) have masses of order $g\alpha$ due to their $D$-term couplings to the flat-direction vev.

We now consider supergravity effects, both in minimal models with soft-breaking terms at the tree level, and in no-scale models [11], where such terms are absent at tree-level but arise from quantum corrections [12]. The superpotential consists of the tree-level MSSM terms and a series of non-renormalizable terms of successively higher dimension, which are induced in the effective theory by the dynamics of whatever is the underlying more fundamental theory. Without imposing $R$ parity (or any other symmetry) all gauge-invariant terms of higher dimension would exist in the superpotential. We may, however, also wish to impose $R$ parity on the higher-dimensional terms, as we have done on the renormalizable interactions, to prevent substantial $R$-parity violation being fed down from high scales by the renormalization-group running of the soft mass terms [13]. If we assume that $R$ parity is a discrete gauge symmetry of the theory, then it would be respected by all gauge-invariant superpotential terms of arbitrary dimension. Relevant higher dimensional superpotential terms which lift the flat direction $\alpha$ are of the form:

$$W \supseteq \lambda_n \frac{\alpha^n}{nM^{n-3}} \tag{5.1}$$

where $\lambda_n$ is a number of order one and $M$ is a large mass scale, e.g., the GUT or Planck scale.

During inflation, supersymmetry is strongly broken by the non-zero energy of the vacuum. In minimal models this is transferred to the observable sector through the Kähler potential at tree level [14], while in no-scale models [11] this happens at the one-loop level [15]. Inflation then induces [14] the soft-breaking terms

$$-C_I {H_I}^2|\alpha|^2 + a\lambda_n H_I \frac{\alpha^n}{nM^{n-3}} + h.c. \tag{5.2}$$

where $C_I$ and $a$ are numbers depending on the sector in which the inflaton lies, and $H_I$ is the Hubble constant during inflation. We shall assume here that $C_I$ is positive and not unnaturally small [14, 15]. In the presence of the $A$ term, the potential along the angular direction has the form $\cos(n\theta + \theta_a)$, where $\theta_a$ is the phase of $a$. Due to its negative mass-squared, the flat direction rolls down towards one of the discrete minima at $n\theta + \theta_a = \pi$ and $|\alpha| = (\frac{C_I}{(n-1)\lambda_n} H_I M^{n-3})^{\frac{1}{n-2}}$, and quickly settles at one of the minima ($\frac{C_I}{(n-1)\lambda_n}$ is $O(1)$). Therefore, at the end of inflation $\alpha$ can be at any of the above-mentioned minima.

In the absence of thermal effects, $\alpha$ would track the instantaneous minimum $|\alpha| \sim (HM^{n-3})^{\frac{1}{n-2}}$ from the end of inflation until the time when $H \simeq m_{\frac{3}{2}}$, where $m_{\frac{3}{2}} \sim$

1 TeV is the low-energy supersymmetry-breaking scale [14]. At $H \simeq m_{\frac{3}{2}}$ the low-energy soft terms

$$m_{\frac{3}{2}}{}^2|\alpha|^2 + A\lambda_n m_{\frac{3}{2}} \frac{\alpha^n}{nM^{n-3}} + h.c. \tag{5.3}$$

would take over, with the mass-squared of $\alpha$ becoming positive, and $\alpha$ would then start its oscillations. Also, the minima along the angular direction would then move in a non-adiabatic way, due generally to different phases for $A$ and $a$. As a result, $\alpha$ starts its free oscillations around the origin with an initial vev $\alpha_{osc.} \sim (m_{\frac{3}{2}} M^{n-3})^{\frac{1}{n-2}}$ and frequency $m_{\frac{3}{2}}$ and, at the same time, the torque exerted on it causes motion along the angular direction. In the case that the flat direction carries a baryon (lepton) number this will lead to a baryon (lepton) asymmetry $n_B$ [7] given by $n_B = \alpha_R \frac{\partial \alpha_I}{\partial t} - \alpha_I \frac{\partial \alpha_R}{\partial t}$. At $m_{\frac{3}{2}} t \gg 1$ the upper bound on $n_B$ [7] may be written

$$n_B \sim \frac{1}{m_{\frac{3}{2}}{}^2 t^2} \alpha_{osc.}{}^3 \left(\frac{\alpha_{osc.}}{M}\right)^{n-3} \tag{5.4}$$

which, after transition to a radiation-dominated universe, results in an $\frac{n_B}{s}$ that remains constant as long as there is no further entropy release.

As we will see in subsequent sections, thermal effects of inflaton decay products with superpotential couplings to the flat direction can fundamentally alter the dynamics of the flat direction oscillation, and necessitate revision of the estimates for the resulting baryon/lepton asymmetries produced.

## 5.3 Flat-Direction Superpotential Couplings and Finite Temperature Effects

As we have seen, the flat direction $\alpha$ has couplings of the form $g^2\alpha^2\phi^2$ to the fields $\phi$ which are in the monomial that represents it. Besides these $D$-term couplings, it also has $F$-term couplings to other fields $\chi$ which are not present in the monomial. These come from renormalizable superpotential Yukawas, and have the form [1]

$$W \supseteq h\alpha\chi\chi \tag{5.5}$$

which results in a term $h^2|\alpha|^2|\chi|^2$ in the scalar potential. Again for illustration, consider the $H^u L$ flat direction: $H^u$ has Yukawa couplings to left-handed and right-handed (s)quarks while $L$ has Yukawa couplings to $H^d$ and right-handed (s)leptons.

In the class of models that we consider, the inflaton is assumed to be in a sector which is coupled to ordinary matter by interactions of gravitational strength only. In this case, the inflaton decay always occurs in the perturbative regime and we need not worry about parametric-resonance decay effects [16]. The inflaton decay rate is $\Gamma_d \sim \frac{m^3}{M_{Pl}{}^2}$, where $m$ is the inflaton mass and $m \leq 10^{13}$ GeV from the COBE data on the CMBR anisotropy [17]. Efficient inflaton decay occurs at the time when $H \simeq \Gamma_d$ and the effective reheat temperature at that time will be $T_R \sim (\Gamma_d M_{Pl})^{\frac{1}{2}}$. For $m \sim 10^{13}$ GeV we get $T_R \sim 10^{10}$ GeV, which is in the allowed range to avoid the gravitino problem [18].

The crucial point to note is that, although inflaton decay effectively completes much later than the start of its oscillation, nonetheless decay occurs throughout this period. In fact, a dilute plasma with temperature $T \lesssim (H\Gamma_d M_{Pl}{}^2)^{\frac{1}{4}}$ (assuming instant thermalization: we address thermalization below) is present from the first several oscillations, until the effective completion of the inflaton decay [19]. It is easily seen that it has the highest instantaneous temperature at the earliest time, which can reach $T \leq 10^{13}$ GeV. This plasma, however, carries a relatively small fraction of the cosmic energy density, with the bulk still in inflaton oscillations. The dilution of relics produced from this plasma by the entropy release from the subsequent decay of the bulk of the inflaton energy is the reason that it does not lead to gravitino overproduction. It is important to note that the energy density in the plasma may be comparable to the energy density stored in the condensate along a flat direction. As a result, the thermal effects from the plasma may affect the dynamics of flat direction evolution which, as we see below, occurs in many cases.

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[1] We note that for $F$-flat directions of the renormalizable piece of the superpotential, which are only lifted by higher-dimensional nonrenormalizable terms, $\alpha$ cannot have such superpotential couplings to $\phi$ fields which appear in the monomial.

All fields with mass less than $T$, and gauge interactions with the plasma particles, can reach thermal equilibrium with the plasma. Those fields which are coupled to the flat direction have generically large masses in the presence of its vev, and might not be excited thermally. These include the $\phi$ fields which are gauge-coupled to $\alpha$ and have a mass $g\alpha$ (up to numerical factors of $O(1)$) and many of the $\chi$ fields which have superpotential couplings to $\alpha$, and hence have a mass $h\alpha$ (also up to numerical factors of $O(1)$). For $g\alpha > T$ or $h\alpha > T$, the former or the latter are not in thermal equilibrium, respectively. We recall that, in the presence of Hubble-induced soft-breaking terms, the minimum of the potential for the flat direction determines that $\alpha \sim (HM^{n-3})^{\frac{1}{n-2}}$, and the plasma temperature is $T \sim (H\Gamma_d {M_{Pl}}^2)^{\frac{1}{4}}$. So a field with a coupling $h$ to the flat direction can be in thermal equilibrium provided that $h\alpha \leq T$, which implies that

$$H^{6-n} \leq \frac{{\Gamma_d}^{n-2}}{h^{4(n-2)}} \frac{{M_{Pl}}^{2(n-2)}}{M^{4(n-3)}} \tag{5.6}$$

and similarly for the gauge coupling $g$.

The back-reaction effect of the plasma of quanta of this field will then induce a mass-squared $+h^2T^2$ for the flat direction to which it is coupled. If this exceeds the negative Hubble-induced mass-squared $-H^2$, the flat direction starts its oscillation. This happens for $hT \geq H$, i.e., for

$$H^3 \leq h^4 \Gamma_d {M_{Pl}}^2 \tag{5.7}$$

and similarly for back-reaction from plasma fields with gauge coupling $g$ to the flat direction. Therefore, a flat direction will start its oscillations if both of the above conditions are satisfied simultaneously. We note that the finite-temperature effects of the plasma can lead to a much earlier oscillatory regime for the flat direction, i.e., when $H \gg m_{3/2}$.

It is clear that, in order for a plasma of the quanta of a field to be produced, the coupling of that field to a flat direction should not be so large that its induced mass prevents its thermal excitation. On the other hand, in order for its thermal plasma to have a significant reaction back on the flat direction, its coupling to the flat direction should not be so small that the thermal mass-squared induced for the flat direction will be smaller than the Hubble-induced contribution. Therefore, to have significant thermal effects, we need couplings of intermediate strength in order to have both conditions simultaneously satisfied. For the fields $\phi$ which have $D$-term couplings of gauge strength $g$ to flat directions, this is usually not the case: As will be seen shortly, in most cases their couplings are too large to satisfy the equilibrium condition. For the fields $\chi$ which have $F$-term couplings of Yukawa strength $h$ to the flat direction, the existence of significant thermal effects depends on the value of $h$, as well as on the initial value of the flat-direction vev $\alpha$, which in turn depends on the mass scale and the dimension of the higher-dimensional operator which lifts the flatness.

To organize our discussion then, we first assess the typical values of $h$ to be expected for couplings to the flat directions. For these typical values, we then estimate the importance of thermal effects on vev's determined by higher-dimensional operators ranging over the various different dimensions that can lift the flat direction, for both the case of lifting by the GUT scale: $O(10^{16})$ GeV, and by the Planck scale: $O(10^{19})$ GeV.

### 5.3.1 Superpotential Couplings of the MSSM Flat Directions

We now list the Yukawa couplings of the MSSM. For low tan$\beta$, the ratio of $H^u$ and $H^d$ vev's, we have

$$\begin{array}{lll} h^u{}_1 \sim 10^{-4} & h^d{}_1 \sim 10^{-5} & h^l{}_1 \sim 10^{-6} \\ h^u{}_2 \sim 10^{-2} & h^d{}_2 \sim 10^{-3} & h^l{}_2 \sim 10^{-3} \\ h^u{}_3 \sim 1 & h^d{}_3 \sim 10^{-2} & h^l{}_3 \sim 10^{-2} \end{array} \tag{5.8}$$

whilst the $h^u$'s and $h^d$'s tend to be more similar for high $\tan\beta$. The only Yukawa couplings which are significantly different from $O(10^{-2})$ are $h^u{}_1$, $h^d{}_1$, $h^l{}_1$, and $h^u{}_3$. Only flat directions which include only the left- and right-handed up squark, the left- and right-handed down squark, the left- and right-handed selectron and the left-handed sneutrino will have an $h$ significantly less than $O(10^{-2})$.

For low $\tan\beta$, any flat direction which includes right-handed top squarks has a Yukawa coupling of $O(1)$ to some $\chi$'s, too large for those $\chi$'s to be in thermal equilibrium, given the expected range of flat direction vev's $\alpha$. The left-handed squarks are coupled to both $H^u$ and $H^d$, so any flat direction which includes a left-handed top squark has a Yukawa coupling of order $10^{-2}$ to $H^d$ as well. For high $\tan\beta$, any flat direction which includes the left- or right-handed top or bottom squarks has a Yukawa coupling of order 1 since the top and bottom Yukawas are of the same order. In general, any flat direction which consists only of the above-mentioned scalars has a Yukawa coupling of order 1 to some $\chi$'s and/or a Yukawa coupling significantly less than $O(10^{-2})$ to other $\chi$'s.

Among all flat directions which are not lifted by the renormalizable superpotential terms, there is only one which allows such a flavor choice: *uude* with one $u$ in the third generation and all other scalars in the first generation (i.e., *tude*). This exceptional flat direction still has a coupling of $O(10^{-4})$ to some $\chi$ fields, since it includes the right-handed up squark. Taking into account all flavor choices for all flat directions which are not lifted at the renormalizable superpotential level, we can use $h \simeq 10^{-2}$ for the coupling of a generic flat direction to $\chi$ fields. For the above-mentioned exceptional case we shall use $h \simeq 10^{-4}$.

So, for our discussion of the dynamics of flat direction oscillations we will consider three representative cases. We will analyze the dynamics when inflaton decay plasmons are coupled to the flat direction by: gauge couplings with coupling $g \simeq 10^{-1}$, generic Yukawa couplings of order $h \simeq 10^{-2}$, or suppressed Yukawa couplings of order $h \simeq 10^{-4}$. Consideration of these cases should allow us to explore the generic range of physical effects that arise in flat direction oscillations, from a plasma of inflaton decay products.

We now undertake a detailed analysis to determine in which cases a plasma of inflaton decay products can be produced, and can initiate the flat-direction oscillations by the reaction they induce on the flat direction. Whether this occurs or not depends on the vev of the flat direction, and the strength of the coupling of the plasma quanta to the flat direction. The initial vev of the flat direction is set by both the underlying scale of the physics of the higher-dimensional operators that lift the flat direction, and, for a given flat direction, by the dimension of the gauge-invariant operator of lowest dimension which can be induced by the underlying dynamics to lift the flat direction.

In order to categorize systematically the various cases which arise, we organize them as follows. First, we divide them into two cases, depending on whether the underlying scale of the new physics responsible for the higher-dimensional operators which lift the flat direction and stabilize the vev at the end of inflation are GUT-scale: $O(10^{16})$ GeV, or Planck-scale: $O(10^{19})$ GeV. Each of these cases is subdivided according to whether the coupling between the flat direction and the inflaton decay products is of gauge strength ($g \simeq 10^{-1}$), standard superpotential Yukawa strength ($h \simeq 10^{-2}$), or exceptional suppressed Yukawa strength ($h \simeq 10^{-4}$). As noted above, this covers the generic range of couplings exhibited by fields in flat directions in the supersymmetric standard model. Finally, each of these cases is subdivided and tabulated according to the dimension of the operator that stabilizes the flat-direction vev, setting (given the possibilities listed above for the underlying scale of the new physics responsible for the operators) the initial vev of the flat direction. These higher-dimensional operators are listed by the order of the monomial in the superfields which appears in the superpotential and is responsible for the operator. We tabulate against the order of the higher-dimensional superpotential term the following quantities (in Planck units[2]): the Hubble constant $H$, the temperature $T$, and the value of the flat-direction vev $\alpha$ at the onset of oscillations, as well as the combination $\frac{hT^2}{H\alpha}$ ($\frac{gT^2}{H\alpha}$ for the case of the gauge coupling) which will be useful when we discuss the produced baryon asymmetry in the next section. We also explain the reasons for the values of the entries appearing, in the light of the two necessary conditions introduced above for inducing the flat-direction oscillations by plasma effects, i.e., that on the one hand the mass of the plasmon induced by the coupling to the flat direction is small enough that it can be populated in the thermal bath from inflaton decay, and, on the other hand, that the coupling is large enough for back-reaction effects from the plasma to lift the flat direction sufficiently to start oscillation despite the effects of the Hubble-induced mass.

### 5.3.2 Plasma Effects for the GUT Scale $M = 10^{16}$ GeV Case

First, let us consider the case that the scale of the new physics that induces operators that stabilize the flat direction is of order the GUT scale: $O(10^{16})$ GeV. We then subdivide this case according to the strength of the coupling of the inflaton decay products to the flat direction. To start, we consider the gauge-coupled case with $g = 10^{-1}$. In this case, it is only for initial flat-direction vev's fixed by either quartic or quintic higher-dimensional terms in the superpotential that the plasma effects can accelerate the onset of flat direction oscillation, with the results shown in Table 5.1. Physically, for superpotential monomials of sixth order or higher, the initial flat-direction vev is sufficiently large that the mass generated by its gauge coupling to the prospective inflaton decay products is large enough to prevent them from being kinematically accessible for thermal excitation. In the case of a quintic superpotential monomial this is also initially the case, and it is only after Hubble expansion has

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[2] From now on, we express some dimensionful quantities in Planck units.

Table 5.1: GUT scale $M = 10^{16}$ GeV, gauge coupling $g = 10^{-1}$

| | $H$ | $T$ | $\alpha$ | $\frac{gT^2}{H\alpha}$ |
|---|---|---|---|---|
| $n = 4$ | $10^{-8}$ | $10^{-\frac{13}{2}}$ | $10^{-\frac{11}{2}}$ | $10^{-\frac{1}{2}}$ |
| $n = 5$ | $10^{-18}$ | $10^{-9}$ | $10^{-8}$ | $10^{7}$ |

reduced $\alpha$, and hence the induced plasmon mass, that thermalized products of inflaton decay can back-react to induce flat-direction oscillation. However, this only occurs for $H < 10^{-16}$, by which time the low-energy soft supersymmetry breaking has already initiated flat-direction oscillation.

For the GUT case $M = 10^{16}$ GeV with generic Yukawa coupling $h = 10^{-2}$, we have the results shown in Table 5.2 for lifting of the flat direction by monomials of the orders listed. In the cases that the order of the monomial is four or five we have no difficulty satisfying the condition that $h\alpha \leq T$, so that they are (thermally) populated in the inflaton decay plasma. For monomials of order six, seven or eight, the induced mass of the prospective plasmon is, in fact, of the same order or slightly larger than the instantaneous effective temperature. So thermally they are present, albeit now with some Boltzman suppression. Moreover, we also note that these induced masses are less than the mass of the decaying inflaton, and so they will be produced in the cascade of inflaton decay products, though, as noted above, after complete thermalization they will be subject to some Boltzmann suppression. In all cases the value of the Hubble constant at the onset of oscillation will be determined by the second condition ($hT \geq H$), which requires that the back-reaction-induced mass overcome the Hubble-induced mass to initiate oscillation. By comparing the results of Tables 5.1 and 5.2, we note that for a general flat direction with $h = 10^{-2}$ which is lifted at the $n = 4$ superpotential level, the values at the onset of oscillations should be taken from the gauge analysis. The reason is that, in this case, the back-reaction of the inflaton decay products which have gauge coupling to the flat direction act at

Table 5.2: GUT scale $M = 10^{16}$ GeV, standard Yukawa coupling $h = 10^{-2}$

| | $H$ | $T$ | $\alpha$ | $\frac{hT^2}{H\alpha}$ |
|---|---|---|---|---|
| $n = 4$ | $10^{-\frac{26}{3}}$ | $10^{-\frac{20}{3}}$ | $10^{-\frac{35}{6}}$ | $10^{-\frac{2}{3}}$ |
| $n = 5$ | $10^{-\frac{26}{3}}$ | $10^{-\frac{20}{3}}$ | $10^{-\frac{44}{9}}$ | $10^{-\frac{16}{9}}$ |
| $n = 6$ | $10^{-9}$ | $10^{-\frac{27}{4}}$ | $10^{-\frac{64}{15}}$ | $10^{-2}$ |
| $n = 7$ | $10^{-\frac{28}{3}}$ | $10^{-\frac{41}{6}}$ | $10^{-\frac{64}{15}}$ | $10^{-\frac{31}{15}}$ |
| $n = 8$ | $10^{-\frac{34}{3}}$ | $10^{-\frac{22}{3}}$ | $10^{-\frac{79}{18}}$ | $10^{-\frac{17}{18}}$ |

an earlier time than the back-reaction of those decay products which have Yukawa couplings to it.

For $M = 10^{16}$ GeV, $h = 10^{-4}$, as a function of the order of the superpotential monomial lifting the flat direction we have the results shown in Table 5.3. For these cases, the flat-direction-induced mass is always less than the instantaneous temperature, due to the weak coupling of the flat direction to the plasmons. The only non-trivial condition now is the second one ($hT \geq H$), which determines how long one must wait before the Hubble-induced mass is sufficiently reduced that the back-reaction-induced flat-direction mass can overcome it to initiate oscillation. This fixes the value of $H$ at the onset of oscillation. Comparing the results of Tables 5.1 and 5.3, we note that for an exceptional flat direction with $h = 10^{-4}$ which is lifted at the $n = 4$ superpotential level, the values at the onset of oscillations should also be taken from the gauge analysis.

### 5.3.3 Plasma Effects for the Planck Scale $M = 10^{19}$ GeV Case

We now turn to the case that the underlying scale of new physics responsible for generating the higher-dimensional operators is the Planck scale. This means that the values of the flat direction vev's after inflation will be larger, raising the mass of

Table 5.3: GUT scale $M = 10^{16}$ GeV, exceptional Yukawa coupling $h = 10^{-4}$

| | $H$ | $T$ | $\alpha$ | $\frac{hT^2}{H\alpha}$ |
|---|---|---|---|---|
| $n = 4$ | $10^{-\frac{34}{3}}$ | $10^{-\frac{22}{3}}$ | $10^{-\frac{43}{6}}$ | $10^{-\frac{1}{2}}$ |
| $n = 5$ | $10^{-\frac{34}{3}}$ | $10^{-\frac{22}{3}}$ | $10^{-\frac{52}{9}}$ | $10^{-\frac{14}{9}}$ |
| $n = 6$ | $10^{-\frac{34}{3}}$ | $10^{-\frac{22}{3}}$ | $10^{-\frac{61}{12}}$ | $10^{-\frac{9}{4}}$ |
| $n = 7$ | $10^{-\frac{34}{3}}$ | $10^{-\frac{22}{3}}$ | $10^{-\frac{14}{3}}$ | $10^{-\frac{8}{3}}$ |
| $n = 8$ | $10^{-\frac{34}{3}}$ | $10^{-\frac{22}{3}}$ | $10^{-\frac{79}{18}}$ | $10^{-\frac{53}{18}}$ |
| $n = 9$ | $10^{-\frac{34}{3}}$ | $10^{-\frac{22}{3}}$ | $10^{-\frac{88}{21}}$ | $10^{-\frac{22}{7}}$ |

prospective plasmons to which they couple, and making it harder to satisfy the constraint that these putative plasmons be generated thermally, or even be kinematically accessible to inflaton decay.

For $M = 10^{19}$ GeV, $g = 10^{-1}$, we have significant effects only for flat directions lifted by superpotential terms arising from quartic or quintic monomials. In all other cases ($n \geq 6$) the flat direction vev is so large that quanta gauge-coupled to it receive sufficiently large masses that they can not be thermally populated at the instantaneous temperature of the inflaton decay products. For the two non-trivial cases we have the results shown in Table 5.4. Only in the $n = 4$ case can we produce thermally a number of plasma quanta sufficient to induce enough mass for the flat direction to initiate its oscillation at an earlier time. In the $n = 5$ case, back-reaction from the plasma of inflaton decay products only manages to induce flat-direction oscillation after $H \ll 10^{-18}$, by which time the low-energy soft supersymmetry breaking has already acted to start the oscillation and also the inflaton decay has been completed.

For $M = 10^{19}$ GeV and $h = 10^{-2}$, we again have a case where flat directions lifted by superpotential monomials of order six or higher result in such a large flat-direction vev that quanta coupled to it receive too large a mass for them to be thermally excited in the plasma of inflaton decay products. For the cases of quartic

Table 5.4: Planck scale $M = 10^{19}$ GeV, gauge coupling $g = 10^{-1}$

| | $H$ | $T$ | $\alpha$ | $\frac{gT^2}{H\alpha}$ |
|---|---|---|---|---|
| $n = 4$ | $10^{-14}$ | $10^{-8}$ | $10^{-7}$ | $10^{4}$ |
| $n = 5$ | $10^{-42}$ | $10^{-15}$ | $10^{-14}$ | $10^{25}$ |

Table 5.5: Planck scale $M = 10^{19}$ GeV, standard Yukawa coupling $h = 10^{-2}$

| | $H$ | $T$ | $\alpha$ | $\frac{hT^2}{H\alpha}$ |
|---|---|---|---|---|
| $n = 4$ | $10^{-10}$ | $10^{-7}$ | $10^{-5}$ | $10^{-1}$ |
| $n = 5$ | $10^{-30}$ | $10^{-12}$ | $10^{-6}$ | $10^{14}$ |

or quintic superpotential monomials, we have the results shown in Table 5.5. We again find that only in the $n = 4$ case can thermal effects actually induce sufficient mass for the flat direction to initiate oscillation earlier. In the $n = 5$ case, back-reaction from the plasma of inflaton decay products only manages to induce flat-direction oscillation after the low-energy soft supersymmetry breaking has already done so, and the inflaton decay has been completed. By comparing the results of Tables 5.4 and 5.5, we note that, for a generic flat direction, i.e., one with $h = 10^{-2}$, the initial values at the onset of oscillations should be taken from the latter. The reason is that, in this case, the back-reaction of the inflaton decay products which have Yukawa couplings to the flat direction act at an earlier time than the back-reaction of those decay products with a gauge coupling to it.

Finally, in the case $M = 10^{19}$ GeV, $h = 10^{-4}$, for flat directions lifted by monomials higher than sixth order the resulting flat-direction vevs are sufficiently large that quanta coupled to it with this coupling are too massive to be excited at the instantaneous temperature of the inflaton decay products. So the nontrivial cases are

Table 5.6: Planck scale $M = 10^{19}$ GeV, exceptional Yukawa coupling $h = 10^{-4}$

| | $H$ | $T$ | $\alpha$ | $\frac{hT^2}{H\alpha}$ |
|---|---|---|---|---|
| $n = 4$ | $10^{-\frac{34}{3}}$ | $10^{-\frac{22}{3}}$ | $10^{-\frac{17}{3}}$ | $10^{-\frac{5}{3}}$ |
| $n = 5$ | $10^{-\frac{34}{3}}$ | $10^{-\frac{22}{3}}$ | $10^{-\frac{34}{9}}$ | $10^{-\frac{32}{9}}$ |
| $n = 6$ | $10^{-\frac{38}{3}}$ | $10^{-\frac{23}{3}}$ | $10^{-\frac{19}{6}}$ | $10^{-\frac{7}{2}}$ |

those in Table 5.6. For $n = 6$, we marginally satisfy the requirement that $hT \simeq \alpha$, necessary for thermal production of quanta coupled to the flat direction, while for $n = 4$ and $n = 5$ we do so comfortably. The second condition, that $hT \geq H$ for effective back-reaction, then serves to determine the value of $H$ at the onset of the thermally-induced oscillation. By comparing the results of Tables 5.4 and 5.6, we note that when the exceptional flat direction with $h = 10^{-4}$ is lifted at the $n = 4$ superpotential level, the values at the onset of oscillations should be taken from the latter. This is because the back-reaction of the inflaton decay products with Yukawa coupling to the flat direction act at an earlier time than the back-reaction of those decay products with gauge couplings to it.

We noted above that thermal effects from the plasma can be important up to $h\alpha \simeq T$ or even somewhat higher. For $\alpha$ less than this, they change the convexity of the effective potential in the $\alpha$ direction at much earlier times, inducing the onset of flat-direction oscillations. We should note that since $\alpha \sim H^{\frac{1}{n-2}}$ and $T \sim H^{\frac{1}{4}}$, then $\alpha$ decreases at the same rate as, or more slowly than, $T$ for $7 \leq n \leq 9$. This means that if $h\alpha \gg T$ right after the end of inflation, it will remain so for later times as well. Therefore, in the $7 \leq n \leq 9$ cases for $M = 10^{19}$ GeV, the Hubble-induced negative mass-squared is dominant and $\alpha$ will not be lifted until $H \simeq 10^{-16}$, if $h\alpha \gg T$ at $H \simeq 10^{-6}$.

In sum, we conclude that for $M = M_{GUT}$, a general flat direction with $h \simeq 10^{-2}$ starts oscillating at $H \gg 10^{-16}$ in the $4 \leq n \leq 8$ cases. For the exceptional one with

$h \simeq 10^{-4}$ it is true in the $n = 9$ case as well. For $M = M_{Planck}$ in the denominator, only in the $n = 4$ case do oscillations of a general flat direction start at $H \gg 10^{-16}$. In the $5 \leq n \leq 9$ cases, the flat direction is protected from thermal effects because its large vev induces such a large mass for fields coupled to it that they cannot be thermally excited in the plasma of inflaton decay products. For the exceptional flat direction with $h \simeq 10^{-4}$, this protection is weaker because of the smaller Yukawa coupling to $\chi$ (which therefore are lighter and can be excited in thermal equilibrium) and, as a result, oscillations start at $H \gg 10^{-16}$ in the $n = 5$, 6 cases also.

We need to elaborate on the implicit assumption that the $\chi$'s ($\phi$'s) are effectively thermal upon production. In the model that we study, the inflaton decays in the perturbative regime, and the decay products have a momentum less than, or comparable to, the inflaton mass $m \sim 10^{-6}$. The $\chi$'s ($\phi$'s) which are produced in two-body decays have a momentum of order $m$ [3]. It can easily be seen that the temperature at which oscillations start (assuming thermal equilibrium) is $\approx 10^{-7}$ in all the above cases. Since the momentum of produced particles is greater than the the average thermal momentum, the dominant process to reach equilibrium is through the decay of $\chi$'s ($\phi$'s) to other particles with smaller momenta. However, the momentum of $\chi$'s ($\phi$'s) is very close to the average thermal momentum. Since thermalization does not change the energy density in the plasma, the number density of $\chi$'s ($\phi$'s) is also close to its thermal distribution. Therefore, the plasma-induced mass-squared $h^2 \frac{n_\chi}{E_\chi}$ ($g^2 \frac{n_\phi}{E_\phi}$) from $\chi$'s ($\phi$'s) is of the same order as $h^2 T^2$ ($g^2 T^2$).

## 5.4 Thermal $A$ Terms and Baryo/Leptogenesis

Motion along the angular direction is required for the build-up of a baryon or lepton asymmetry. This is possible if a torque is exerted on $\alpha$ or, equivalently, if $\alpha$ is not in one of the discrete minima along the angular direction, when it starts oscillating. These discrete minima are due to the $A$ term part of the potential. Before the start of oscillations, the Hubble-induced $A$ terms are dominant, and the locations of the minima are determined by them. During inflation, $\alpha$ rolls down towards one of these minima and rapidly settles there. After inflation it tracks that minimum and there is no motion along the angular direction [14]. What is necessary then is a non-adiabatic change in the location of the minima, such that at the onset of oscillations $\alpha$ is no longer in a minimum along the angular direction. In the absence of thermal effects, $\alpha$ would start its angular motion (as well as its linear oscillations) at $H \simeq m_{\frac{3}{2}}$. This occurs as a result of uncorrelated phases of the $A$ terms induced by the Hubble expansion and low-energy supersymmetry breaking. At this time, the latter takes over from the former, and $\alpha$ will in general no longer be in a minimum along the angular direction. This will lead to the generation of a baryon or lepton asymmetry if $\alpha$ carries a non-zero number of either [14].

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[3] The $\phi$'s generically have larger $\alpha$-induced masses than do the $\chi$'s, so their production may be delayed until $\alpha$ Hubble-dilutes to a smaller value.

As we have seen above, due to thermal effects, in many cases the flat directions start oscillating at much larger $H$. At this time the Hubble-induced $A$ terms are still much larger than the low-energy ones from hidden-sector supersymmetry breaking. In order to have angular motion for $\alpha$, another $A$ term of size comparable to the Hubble-induced one, but with uncorrelated phase, is required. Since it is finite-temperature effects from the plasma that produce a mass-squared which dominates the Hubble-induced one, one might expect that the same effects also produce an $A$ term which dominates the Hubble-induced $A$ term. This is the only new effect that could produce such an $A$ term with uncorrelated phase, as the thermal plasma is the only difference from the standard scenario.

The simplest such thermal $A$ terms arise at tree-level from cross terms from the following two terms in superpotential

$$h\alpha\chi\chi + \lambda_n \frac{\alpha^n}{nM^{n-3}} \tag{5.9}$$

which results in the contribution

$$h\lambda_n \frac{\chi^{*2}\alpha^{n-1}}{M^{n-3}} + h.c. \tag{5.10}$$

in the scalar potential. In thermal equilibrium, $< \chi^{*2} >$ can be approximated by $T^2$ and therefore the thermal $A$ term is of order

$$h\lambda_n \frac{T^2\alpha^{n-1}}{M^{n-3}} \tag{5.11}$$

There is another thermal $A$ term that arises from one-loop diagrams with gauginos and fermionic partners of $\alpha$. It results in a contribution, in the thermal bath, of order:

$$\lambda_n \left(\frac{gT}{4\pi\alpha}\right)^2 \frac{T\alpha^n}{M^{n-3}} \tag{5.12}$$

We have checked that, for the parameter range of interest for this process, this has the same order of magnitude as the tree-level $A$ term. In the following, we use the tree-level term for our estimates.

The ratio of the thermal $A$ term to the Hubble-induced one is $\frac{hT^2}{H\alpha}$. It is clear from the results summarized in the Tables that, at the onset of $\alpha$ oscillations, the thermal $A$ term is weaker than the Hubble-induced one in all cases. Therefore, at this time the minima along the angular direction are slightly shifted, the curvature at the minima is still determined by the Hubble-induced $A$ term, and the force in the angular direction is of order $\lambda_n \frac{hT^2\alpha^{n-2}}{M^{n-3}}$. The ratio of the thermal $A$ term to the Hubble-induced one, however, grows as $\frac{hT^2}{H\alpha}$ increases in time. We will keep both $A$ terms in the equation of motion of $\alpha$ in what follows.

Let us consider the case where oscillations start because of the back-reaction of the $\chi$'s, as this is the most common case. The mass of $\alpha_R$ and $\alpha_I$ will then be of order $hT$ [4]. The equation of motion for the flat direction will then be:

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[4] We shall use $gT$ for the plasma-induced mass if oscillations start because of the back-reaction of the $\phi$'s.

$$\ddot{\alpha}+3H\dot{\alpha}+h^2T^2\alpha+(n-1)h\lambda_n\frac{T^2\alpha^{*n-2}}{M^{n-3}}+A\lambda_n\frac{H\alpha^{*n-1}}{M^{n-3}}+(n-1)\lambda_n{}^2\frac{|\alpha|^{2(n-2)}}{M^{2(n-3)}}\alpha=0 \quad (5.13)$$

At this time the universe is in a matter-dominated phase (by the oscillating inflaton field) and thus $H = \frac{2}{3t}$. Also, $T^2 = (H\Gamma_d M_{Pl}{}^2)^{\frac{1}{2}} \sim t^{-\frac{1}{2}}$ for $H \geq 10^{-18}$. After re-scaling $\alpha \rightarrow (\frac{H_{osc.}M^{n-3}}{\lambda_n})^{\frac{1}{n-2}}\alpha$ and $t \rightarrow H_{osc.}t$ ($H_{osc.}$ is the Hubble constant at the onset of oscillations), we get the following equations of motion for the real and imaginary components of $\alpha$

$$\ddot{\alpha}_R+\tfrac{2}{t}\dot{\alpha}_R+a\frac{\alpha_R}{t^{\frac{1}{2}}}+b\frac{|\alpha|^{n-2}}{t^{\frac{1}{2}}}\cos((n-2)\theta+\varphi)+A\frac{|\alpha|^{n-1}}{t}\cos((n-1)\theta)+(n-1)|\alpha|^{2(n-2)}\alpha_R=0$$

$$\ddot{\alpha}_I+\tfrac{2}{t}\dot{\alpha}_I+a\frac{\alpha_I}{t^{\frac{1}{2}}}-b\frac{|\alpha|^{n-2}}{t^{\frac{1}{2}}}\sin((n-2)\theta+\varphi)-A\frac{|\alpha|^{n-1}}{t}\sin((n-1)\theta)+(n-1)|\alpha|^{2(n-2)}\alpha_I=0 \quad (5.14)$$

Here $a = \frac{h^2T_{osc.}{}^2}{H_{osc.}{}^2}$ ($T_{osc.} = (H_{osc.}\Gamma_d M_{Pl}{}^2)^{\frac{1}{2}}$ is the plasma temperature at the onset of oscillations), $b = (n-1)a\lambda_n\frac{(\frac{H_{osc.}M^{n-3}}{\lambda_n})^{\frac{n-3}{n-2}}}{M^{n-3}}$, and $\varphi$ is the relative phase (of $O(1)$) between the thermal $A$ term and the Hubble-induced $A$ term.

The first two terms plus the superpotential one in these equations are the same as the ones in the equations derived in [14]. However there are some important differences. First of all, the flat direction mass-squared is not the (constant) low-energy value $m_{\frac{3}{2}}{}^2$ but the thermal mass which is redshifted as $t^{-\frac{1}{2}}$. Also, the Hubble-induced $A$ term with coefficient $H$ appears instead of the low-energy one with coefficient $m_{\frac{3}{2}}$ (the second one is negligible for $H \gg 10^{-16}$). This explains the $\frac{1}{t}$ factor in front of the Hubble induced $A$ term. Finally, another $A$ term, the thermal one, appears which, because of its $T^2$ dependence, is also redshifted as $t^{-\frac{1}{2}}$.

At the onset of oscillations $t = t_i = \frac{2}{3}$ and $\alpha$ is in one of the minima which are determined by the Hubble soft terms. Therefore, recall that $|\alpha|_i$ which was $(\frac{H_{osc.}M^{n-3}}{\lambda_n})^{\frac{1}{n-2}}$ before re-scaling, is scaled to $|\alpha|_i = 1$, and also $\theta_i = n\pi$. For $\theta_i = 0$,

Table 5.7: The value of $\frac{n_B}{s}$ for flat directions which undergo plasma-induced oscillations

| | $M = 10^{16}$ GeV | | $M = 10^{19}$ GeV | |
|---|---|---|---|---|
| | $h = 10^{-2}$ | $h = 10^{-4}$ | $h = 10^{-2}$ | $h = 10^{-4}$ |
| $n = 4$ | $< 10^{-11}$ | $< 10^{-11}$ | $< 10^{-11}$ | $3 \times 10^{-11}$ |
| $n = 5$ | $< 10^{-11}$ | $3 \times 10^{-11}$ | no plasma effect | $3 \times 10^{-9}$ |
| $n = 6$ | $10^{-11}$ | $4 \times 10^{-11}$ | no plasma effect | $10^{-7}$ |
| $n = 7$ | $4 \times 10^{-11}$ | $10^{-10}$ | no plasma effect | no plasma effect |
| $n = 8$ | $10^{-10}$ | $3 \times 10^{-10}$ | no plasma effect | no plasma effect |
| $n = 9$ | no plasma effect | $5 \times 10^{-10}$ | no plasma effect | no plasma effect |

$A = (n-1)$, and $\lambda_n = 1$, we have solved these equations numerically and calculated $n_B = \alpha_R \frac{\partial \alpha_I}{\partial t} - \alpha_I \frac{\partial \alpha_R}{\partial I}$. We find that among the cases listed in the Tables 5.1-5.6, only for the ones listed in Table 5.7 do we get an $\frac{n_B}{s}$ of order $10^{-11}$ or larger, before any subsequent (after reheating) dilution.

It is seen that in some cases $\frac{n_B}{s}$ is near the observed value of $5 \times 10^{-10}$. However in the most general case, when the standard model gauge group is the only symmetry group, these viable flat directions constitute only a small subset of all flat directions. It is also seen that $\frac{n_B}{s}$ is larger for the exceptional flat direction, for the $M = 10^{19}$ GeV cases, and for the flat directions which are lifted at a higher level $n$. It is easily understandable as for larger $M$ and $n$, and for smaller $h$, plasma-induced oscillations start later and closer to the efficient reheat epoch $H = 10^{-18}$. Larger $M$ and $n$ lead to a larger vev for the flat direction and, therefore, the condition $h\alpha \leq T$ will be satisfied at a later time. A smaller $h$ on the other hand implies that the condition $hT \geq H$ will be satisfied at a later time. Later oscillations mean less dilution of the generated lepton/baryon asymmetry by the plasma of inflaton decay products (recall that $s \sim T^3$ is redshifted only as $t^{-\frac{3}{4}}$ for $H \geq 10^{-18}$).

Now we should comment how our results are affected by changes in the model-dependent constants involved in the calculations: the reheat temperature $T_R$ (or equivalently the inflaton decay rate), and the constant $\frac{C_I}{(n-1)\lambda_n}$ which appears in the expression for the flat-direction vev. There are two concerns in this regard. First, whether the two conditions for plasma-induced $\alpha$ oscillations still result in a consistent value for $H_{osc.}$ which is greater than $10^{-16}$, and, secondly, what the corresponding change in the estimated value for $\frac{n_B}{s}$ will be. In our calculations we used $T_R \simeq 10^{-9}$ and $\frac{C_I}{(n-1)\lambda_n} \simeq 1$. If we assume instead that $T_R \simeq 10^{-10}$ and $\frac{C_I}{(n-1)\lambda_n} \simeq 10^{-1}$, it turns out that for all cases except the marginal ones ($n = 6, 7, 8$ cases for $h = 10^{-2}$ and $M = 10^{16}$ GeV and the $n = 6$ case for $h = 10^{-4}$ and $M = 10^{19}$ GeV), plasma effects still trigger the oscillations for $H \geq 10^{-16}$, though at a somewhat smaller $H$. Moreover, the value of $\frac{n_B}{s}$ remains within the same order of magnitude. Therefore, the plasma-induced oscillations of the (non-marginal) flat directions, and the resulting value of $\frac{n_B}{s}$, are rather insensitive to the exact order of magnitude of $T_R$ and $\frac{C_I}{(n-1)\lambda_n}$.

## 5.5 Evaporation of the Flat Direction

Now let us find the time when the $\alpha$ condensates are knocked out of the zero mode by the thermal bath. For the evaporation to happen, it is necessary that the thermal bath includes those particles which are coupled to $\alpha$. Then, two conditions should be satisfied: first, the scattering rate of $\alpha$ off the thermal bath must be sufficient for equilibriation, and secondly, the energy density in the bath must be greater than that in the condensate. The flat direction has couplings both of Yukawa strength $h$ to $\chi$'s and of gauge strength $g$ to $\phi$'s. The conditions for thermal production of $\chi$'s and $\phi$'s are $h\alpha \leq T$ and $g\alpha \leq T$, respectively. Since $h < g$, the $\chi$'s will come to thermal equilibrium at an earlier time. On the other hand, the scattering rate of $\alpha$ off the thermal $\chi$'s is $\Gamma_{scatt.} \sim h^4 T$ while the rate for scattering of $\alpha$ off the thermal $\phi$'s is $\Gamma_{scatt.} \sim g^4 T$. Therefore, $\chi$'s are produced earlier but in general have a smaller scattering rate. The competition between the $\chi$'s and $\phi$'s, and between the ratio of the energy density of the flat direction to the energy density in the plasma will determine whether and how the flat direction evaporates.

First we consider those flat directions which have plasma-induced oscillations. If oscillations start due to the back-reaction of $\chi$'s (which is the situation for most cases) $\Gamma_{scatt.} \sim h^4 T$. For a general flat direction with $h \simeq 10^{-2}$, this is comparable to $H$ at $H \simeq 10^{-17}$, while for the exceptional flat direction with $h \simeq 10^{-4}$ this occurs at a much smaller $H$. However, after $\alpha$ starts its oscillation, it is redshifted as $t^{-\frac{7}{8}}$ while $T$ is redshifted as $t^{-\frac{1}{4}}$. This implies that $\frac{g\alpha}{T}$ decreases rapidly and soon the $\phi$'s will be in thermal equilibrium. The rate for scattering of $\alpha$ off thermal $\phi$'s is $\Gamma_{scatt.} \sim g^4 T$ and $\Gamma_{scatt.} \geq H$ at $H \leq 10^{-12}$. The energy density in the condensate at the onset of oscillations is $h^2\alpha^2T^2 \leq T^4$ (recall that $h\alpha \leq T$ at this time). The ratio of the two energy densities is further redshifted as $t^{-\frac{5}{4}}$ (for $H \geq 10^{-18}$) which ensures the second necessary condition for the evaporation of condensate, i.e., that the plasma energy density is dominant over the energy density in the condensate. It can easily be checked that the condensate evaporates at $H \gg 10^{-18}$, before the inflaton decay is completed [5].

In those cases in which the plasma effects do not lead to an early oscillation of the flat direction, oscillations start at $H \simeq 10^{-16}$, when the low-energy supersymmetry breaking takes over the Hubble-induced one. It is important to find the time when the condensate will evaporate in these cases too. For such flat directions, the ratio of the baryon number density to the condensate density is of order one [14]. Therefore, if the condensate dominates the energy density of the universe before evaporation, the resulting $\frac{n_B}{s}$ will also be of order one. Some regulating mechanism is then needed in order to obtain the value for successful big bang nucleosynthesis: $\frac{n_B}{s} \sim 10^{-10}$ [20].

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[5] If $\alpha$ oscillations start due to the back-reaction of $\phi$'s, from the beginning $g\alpha \leq T$ and $\Gamma_{scatt.} \sim g^4 T$. Therefore, there is no need to wait for further redshift of $\alpha$ and again the condensate evaporates at $H \gg 10^{-18}$.

Now consider a general flat direction with $h \simeq 10^{-2}$. As we showed, in the $5 \leq n \leq 9$ cases for $M = 10^{19}$ GeV, and the $n = 9$ case for $M = 10^{16}$ GeV, plasma effects are not important and the flat direction starts oscillating at $H \simeq 10^{-16}$. By $H \simeq 10^{-18}$ the inflaton has efficiently decayed and $\alpha$ has been redshifted by a factor of $10^{-2}$. From then on, the universe is radiation-dominated, so $\alpha \propto t^{-\frac{3}{4}}$ and $T \propto t^{-\frac{1}{2}}$. Therefore, the energy density in the condensate is redshifted as $t^{-\frac{3}{2}}$ whilst the energy density in radiation is redshifted as $t^{-2}$. If the condensate does not evaporate (or decay) until very late times, its energy density dominates that of the radiation and universe will again be matter-dominated. At the beginning of oscillations, i.e., at $H \simeq 10^{-16}$, $\alpha$ has the largest vev in the $n = 9$ case for $M = 10^{19}$ GeV, which is $\alpha \simeq 10^{-\frac{16}{7}}$. At $H \simeq 10^{-18}$ this is redshifted to $\alpha \simeq 10^{-\frac{30}{7}}$ which still leaves $h\alpha > T$, so plasmons with this Yukawa coupling to the flat direction cannot be produced. However, since $\alpha$ redshifts more rapidly than $T$, eventually $h\alpha$ becomes of order $T$, after a time such that

$$T \simeq 10^{-\frac{101}{7}}, \ \alpha \simeq 10^{-\frac{87}{7}} \tag{5.15}$$

It is easily seen that at this time the energy density in the condensate and in the radiation are of the same order. Moreover, $\Gamma_{scatt.} \sim 10^{-8} T \gg H$ and the condensate evaporates promptly. This case is marginal as the condensate almost dominates the energy density of the universe at evaporation.

In the $5 \leq n \leq 8$ cases for $M = 10^{19}$ GeV and the $n = 9$ case for $M = 10^{16}$ GeV, the vev is considerably smaller and the energy density in radiation is even more dominant. Therefore, a general flat direction with $h \simeq 10^{-2}$ will evaporate before dominating the energy density of the universe. We summarize the situation for a general flat direction with $h \simeq 10^{-2}$, regarding both the early, i.e., plasma-induced, oscillation, and evaporation, in Table 5.8.

For the exceptional flat direction with $h \simeq 10^{-4}$ the situation is different. Here plasma effects are not important in the $7 \leq n \leq 9$ cases for $M = 10^{19}$ GeV. In the

Table 5.8: Viability of scenarios with generic Yukawa coupling $h = 10^{-2}$

| | $M = 10^{16}$ GeV | | $M = 10^{19}$ GeV | |
|---|---|---|---|---|
| | Early Oscillation | Evaporation | Early Oscillation | Evaporation |
| $n = 4$ | ✓ | ✓ | ✓ | ✓ |
| $n = 5$ | ✓ | ✓ | | ✓ |
| $n = 6$ | marginal | ✓ | | ✓ |
| $n = 7$ | marginal | ✓ | | ✓ |
| $n = 8$ | marginal | ✓ | | ✓ |
| $n = 9$ | | ✓ | | marginal |

$n = 9$ case the condition for thermal production of $\chi$'s, $h\alpha = T$ gives

$$T \simeq 10^{-\frac{73}{7}}, \ \alpha \simeq 10^{-\frac{43}{7}} \tag{5.16}$$

which means we do not need as much redshift to reduce $\alpha$, so $\chi$'s are produced earlier and at a higher temperature. However, $\Gamma_{scatt.} \sim 10^{-16}T$, which is much smaller than $H$ at this time. Therefore, the condensate cannot evaporate by scattering off the $\chi$'s. It is easily seen that $hT > m_{\frac{3}{2}} \simeq 10^{-16}$ when $h\alpha = T$. This implies that the mass and energy density of the flat direction are $hT$ and $h^2\alpha^2T^2$, respectively, upon thermal production of $\chi$'s, and the energy density in the flat direction and the thermal bath are comparable. As long as $hT \geq m_{\frac{3}{2}}$, $\alpha$ and $T$ are both redshifted as $t^{-\frac{1}{2}}$. During this interval $\frac{\alpha}{T}$ remains constant and the flat direction and plasma energy densities remain comparable. Later, when $T < 10^{-12}$ we have $hT < m_{\frac{3}{2}}$, and the energy density in the condensate is $m_{\frac{3}{2}}{}^2\alpha^2$ and begins to dominate the thermal energy density. At some point, $g\alpha < T$ and $\phi$'s can be produced thermally. The scattering rate of the condensate off the $\phi$'s is $\Gamma_{scatt.} \sim 10^{-4}T$ which is clearly at equilibrium. However, the energy density in the condensate is now overwhelmingly dominant and evaporation

Table 5.9: Viability of scenarios with exceptional Yukawa coupling $h = 10^{-4}$

| | $M = 10^{16}$ GeV | | $M = 10^{19}$ GeV | |
|---|---|---|---|---|
| | Early Oscillation | Evaporation | Early Oscillation | Evaporation |
| $n = 4$ | ✓ | ✓ | ✓ | ✓ |
| $n = 5$ | ✓ | ✓ | ✓ | ✓ |
| $n = 6$ | ✓ | ✓ | marginal | ✓ |
| $n = 7$ | ✓ | ✓ | | |
| $n = 8$ | ✓ | ✓ | | |
| $n = 9$ | ✓ | ✓ | | |

does not occur. For the $n = 7$, 8 cases the situation is similar and the condensate does not evaporate. The summary for the exceptional flat direction, regarding both the early, i.e., plasma-induced, oscillation, and evaporation, is illustrated in Table 5.9.

In summary: a general flat direction, i.e., with $h \simeq 10^{-2}$, which does not have plasma-induced early oscillation, does not come to dominate the energy density of the universe (the $n = 9$ case for $M = 10^{19}$ GeV is marginal). For the exceptional flat direction, i.e., with $h \simeq 10^{-4}$, the situation is different, and it dominates the energy density of the universe before decay.

## 5.6 Discussion

We have found that all flat directions, except those which are lifted by nonrenormalizable superpotential terms of high dimension and with a large mass scale in the denominator, start oscillating at early times due to plasma effects. For a general flat direction with $h \simeq 10^{-2}$ these are the $n = 4$ case for $M = 10^{19}$ GeV and the $4 \leq n \leq 8$ cases for $M = 10^{16}$ GeV (with the $6 \leq n \leq 8$ cases being marginal and sensitive to model-dependent parameters). For the exceptional flat direction with $h \simeq 10^{-4}$ these are the $4 \leq n \leq 6$ cases for $M = 10^{19}$ GeV (with the $n = 6$ case being marginal and sensitive to model-dependent parameters) and all $n$ for $M = 10^{16}$ GeV. In these cases it is difficult to achieve efficient baryon asymmetry generation by the oscillation of the condensate along the flat direction. We showed that a general flat direction, i.e., one with $h \simeq 10^{-2}$, which is not lifted by thermal effects, still evaporates before dominating the energy density of the universe. This is not important for baryogenesis, however, and the resulting dilution by the thermal bath can be used to regulate the $\frac{n_B}{s}$ which is initially of order one. On the other hand, the exceptional flat direction, i.e., one with $h \simeq 10^{-4}$, which is not lifted by plasma effects, dominates the energy density of the universe before its decay.

For models with supersymmetry breaking via low-energy gauge mediation, on the other hand, the evaporation of the condensate has yet another implication. In such models there is a candidate for cold dark matter, the so called Q-ball [21]. In order to have stable Q-balls as dark matter candidates, some flat directions must dominate the energy density of the universe. This means that any flat direction which is evaporated by the thermal bath cannot be used to form a Q-ball.

Now the question is which flat directions are lifted by $n > 4$ terms. A look at [10] reveals that only 18 out of 295 directions which are $D$- and $F$-flat at the renormalizable level in the MSSM are not lifted at the $n = 4$ level. Even a smaller subset of only 2 flat directions are not lifted at the $n = 6$ level. If nonrenormalizable terms of dimension 4 and 5 are not forbidden by imposing other symmetries, only a very few flat directions in the MSSM can be used for baryogenesis and even fewer for Q-ball formation (regardless of the mass scale in the denominator or Yukawa couplings of these flat directions). This is if all higher-order terms which respect gauge symmetry exist in the superpotential. With other symmetries (discrete or continuous) imposed on the model, a specific flat direction will, in general, be lifted at a higher level. The initial vev of $\alpha$ can then be larger and $\chi$ and $\phi$ quanta may not be produced thermally, and the standard treatment of the Affleck-Dine baryogenesis may be valid.

Model-dependent analysis is needed to identify at which level a given flat direction is actually lifted, in a given model.

Finally, an interesting possibility is the parametric-resonance decay of a supersymmetric flat direction to the fields $\phi$ to which it is gauge-coupled. The occurrence and implications of a potential parametric resonance are more pronounced for those flat directions which start their oscillations at $H \simeq 10^{-16}$, as in the standard scenario. They have an incredibly large $q = (\frac{g\alpha}{2m_{\frac{3}{2}}})^2$ [6] which could be as large as $O(10^{20})$ (the parameter $q$ determines the strength of resonance [16]). Explosive resonance decay could also prevent these flat directions from dominating the energy density of the universe. However, the situation is too complicated to allow simple estimates based on the results of parametric-resonance decay of a real scalar field. First of all, the renormalizable part of the scalar potential (including the $D$-term part which is responsible for parametric-resonance decay to $\phi$'s) is fully known and very complicated. Moreover, the flat direction itself is a complex scalar field. This may result in out-of-phase oscillations in the imaginary part of the flat direction, as well as in other scalar fields which are coupled to the same $\phi$, which can then substantially alter the outcome of simple parametric resonance [22].

## 5.7 Conclusion

In conclusion, we have seen that many of the MSSM flat directions may start their oscillations differently than in the standard scenario, where the low-energy supersymmetry breaking determines the onset of oscillations. The two key ingredients for such a different behaviour are: superpotential Yukawa couplings of the flat directions to other fields, and the thermal plasma from partial inflaton decay, whose instantaneous temperature is higher than the reheat temperature. Together, these lead to an earlier start of the oscillations. On the one hand, the masses of those fields which are coupled to the flat direction that are induced by the flat-direction vev are then small enough to be kinematically accessible to inflaton decay and, on the other hand, induce large enough thermal masses for flat directions from the back-reaction of those fields to overcome the negative Hubble-induced mass-squared of the flat directions. Subsequently, thermal masses and $A$ terms may be responsible for baryo/leptogenesis, but typically result in an insufficient baryon/lepton asymmetry of the universe. The oscillations are also terminated earlier, due to evaporation of the flat direction through its interactions with the thermal plasma. It was also shown that even for many flat directions whose oscillations are not initiated by plasma effects, these effects cause them to evaporate before dominating the energy density of the universe.

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[6] For those flat directions which have plasma-induced oscillations, $m_{\frac{3}{2}}$ is replaced by $hT$ or $gT$, leading to a considerably smaller $q$.

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# Chapter 6

# Conclusion

With the four case studies completed, some final statements are in order. The work presented in this thesis was mainly based on model-independent investigations (except that of chapter two), yet some definitive conclusions can be reached for individual chapters and the overall work.

Let us first consider the individual chapters. In chapter 2, we saw that R-parity violation in the heavy neutrino sector leads to instability of the LSP through induced soft supersymmetry breaking masses at low energies. The LSP decay rate, however, crucially depends on the type of R-parity violating term. For the $NNN$ coupling it is suppressed by the mass of the right-handed neutrino. This implies that with a large enough mass for successful implementation of the see-saw mechanism, the LSP could still be of cosmological relevance. For the $NH_1H_2$ coupling, however, the situation is rather different. In this case the decay rate is not suppressed by heavy masses leading, therefore, to rapid decay of the LSP unless the $NH_2H_2$ coupling is unnaturally small. This investigation proves the power of cosmology, in particular when combined with laboratory limits, in constraining new physics.

We learned from chapter 3 that self-interactions of bosonic decay products may qualitatively alter the nature of parametric resonance. Although the original work was conducted in the narrow-band resonance regime, later studies proved the generality of its results in the broad-band case as well. The main motivation in this study was gravitino overproduction (in supersymmetric models) as a result of the parametric resonance decay of the inflaton. It was shown that self-interactions of moderate strength can prevent the abrupt reheat that leads to disastrous gravitino production, and that supersymmetry can naturally provide such self-interactions of gauge strength. Model-dependent analysis is needed in different cases, to see whether the decay products have self-interactions of this kind.

The main conclusion of chapter 4 is that for a complex scalar field the broad-band resonance may be delayed until later times. With a phase invariant coupling, a (not unnaturally small) phase difference in oscillations of the real and imaginary parts can shut-off the resonance at early stages. This delay has very important consequences. It may solve the gravitino problem, since a later decay leads to a lower reheat temperature. It also has very important consequences for superheavy particle production during resonance. Such a phase difference can be important even when the decay products have a relatively small occupation number, as its effect is to induce a time-independent mass term for the decay products, which may restore the adiabaticity in the time-variation of their masses. This is most effective during the first oscillations of the field; therefore, resonance can be killed very early. The interesting point is that the strength of this out of phase component is independent of the couplings of the oscillating field to other fields. However, model-dependent studies are needed to determine its strength in each case. The same results can hold if several real oscillating fields are coupled to the same final state field.

The studies in chapters 3 and 4 convey two main messages. First, broad-band resonance is very sensitive to the behaviour of oscillating field. All that is needed for broad-band resonance is an interval where the time-variation of the oscillating field is non-adiabatic. The number density of produced particles and their spectrum depends only on the behaviour of the oscillating field in this interval. In general, this is very short compared with the period of oscillations, implying that even a very small distortion in the oscillations can violate this non-adiabaticity and destroy resonance. Second, the time has come for model-dependent analyses. During the past years, the dynamics of parametric resonance has been understood to a very good extent. To make quantitative predictions, different models should be (re)considered in order to see whether parametric resonance indeed plays a role in them.

The conclusion of chapter 5 is that general estimates for the baryon asymmetry of the universe from Affleck-Dine mechanism may need substantial modifications. It is generally believed that supersymmetric flat directions start their oscillations when low energy supersymmetry breaking becomes dominant over supersymmetry breaking effects by non-zero energy of the early universe. However, it was shown that for many flat directions supersymmetry breaking effects induced by the plasma of partial inflaton decay lead to a much earlier oscillation. Most notably, when all higher dimensional terms are allowed, flat directions which are lifted at the $n = 4$, 5 levels (and consist almost all of the MSSM flat directions) start their oscillations due to such plasma effects, regardless of the underlying scale of new physics. The estimates for the baryon asymmetry generation from such flat directions are typically very small. They could be substantially larger if these flat directions are lifted at a higher superpotential level. This would be the case if other symmetries which forbid the presence of the relevant non-renormalizable superpotential terms are imposed. Therefore, in this case too, model-dependent studies are needed to make clear predictions.

In conclusion, the start and termination of scalar field oscillations in the early universe are generally much more complicated than previously thought. This confronts us with new challenges in model building. Consequently, some previously viable models may now be ruled out. It is very important, therefore, to (re)consider each model and see whether it needs modification. Today we know much more about reheating and baryogenesis (and their inter-relations) than 5 years ago, but a realistic model with consistent and acceptable predictions is still lacking. This ensures that astro-particle physics will remain as active and interesting as ever in the coming years.